

# Type Dependence and Scale Dependence in Top Wealth Concentration

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## Abstract

In random growth models, top wealth concentration arises from type dependence, a persistent type that compounds wealth faster, or scale dependence, growth depending on current wealth alone. Quantitative models do not identify their relative importance from wealth data, and wealth inequality and one-period wealth mobility cannot distinguish them. Wealth over several periods distinguishes them under i.i.d. labor income, but persistent labor income reproduces the shape of a permanent type's wealth mobility profile. I fit the model to the 2022 U.S. top 10 wealth share and to the dispersion of PSID wealth growth. Where it matches PSID wealth mobility, scale dependence accounts for the larger share of top wealth concentration unless the type is very persistent and wide, where type dependence accounts for 41 to 56 percent. At the narrow, persistent types of most existing calibrations, the fitted model attributes at most 18 percent to type dependence.

**JEL classification:** C23, D31, D63, E21

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# 1 Introduction

The wealthiest 10 percent in the United States hold between 69 and 77 percent of aggregate wealth (Saez and Zucman, 2016; Kuhn, Schularick and Steins, 2020; Smith, Zidar and Zwick, 2023). The same wealth concentration is consistent with two worlds. In one, some households hold a permanent advantage and pull away (type dependence). In the other, no advantage lasts, and a household's growth depends only on its current wealth and on luck (scale dependence). The two worlds fit the same wealth distribution yet tell opposite stories about opportunity.

The two worlds differ only in whether the growth factor has memory. In a random growth model of wealth, next period's wealth is current wealth times a growth factor, plus labor income. Under scale dependence the growth factor is drawn afresh each period, given current wealth. Under type dependence it contains a persistent household type, so past wealth predicts future wealth beyond current wealth. Let  $\rho$  be the persistence of the type: scale dependence is the memoryless case  $\rho = 0$ , and type dependence is any  $\rho > 0$ .

The relative importance of type dependence and scale dependence matters for policy. The welfare-maximizing marginal wealth tax rate is  $-0.88$  percent under pure scale dependence and  $2.41$  percent under pure type dependence (Gaillard and Wangner, 2023). Replacing the capital income tax with a wealth tax raises average welfare when returns are heterogeneous and persistent (Guvenen et al., 2023). Yet the quantitative models of the wealth distribution do not identify this relative importance from wealth data. They assume the persistence of the type (Hubmer, Krusell and Smith, 2021) or calibrate it to how often households enter and leave equity investment or entrepreneurship (Gaillard and Wangner, 2023; Gaillard et al., 2024).

**Results** I obtain three theoretical results and one quantitative result. First, wealth inequality and one-period wealth mobility contain no information about the relative importance of type dependence and scale dependence, in any model class that contains the memoryless twin. Every type-dependent economy has a memoryless twin: a Markov chain in wealth with the same stationary wealth distribution and the same one-step transition. With i.i.d. labor income the memoryless twin is a scale-dependent economy.

Second, with i.i.d. labor income, wealth over several periods distinguishes a persistent type from its memoryless twin. A persistent type makes wealth non-Markov, so the joint law of wealth at three consecutive dates differs between the two at every  $\rho > 0$ . The wealth mobility profile traces the correlation between a household's wealth today and its wealth  $k$  periods

later over the horizon  $k$ . With a permanent type it converges to a positive limit, and in the memoryless twin it converges to zero. With two type values and a growth factor equal to the type, the wealth mobility profile also identifies the type's persistence locally, at almost every parameter value.

Third, persistent labor income reproduces the shape of a permanent type's wealth mobility profile. In a scale-dependent economy with i.i.d. growth factors, a permanent income level keeps the wealth mobility profile positive at every horizon, as a permanent type does, and the wealth mobility profile converges geometrically to that floor. A positive floor in the wealth mobility profile therefore no longer identifies a type.

The quantitative result is that scale dependence accounts for the larger share of U.S. top wealth concentration unless the type is both very persistent and wide. I fit the random growth model to the 2022 U.S. top 10 wealth share across combinations of the type's persistence and its width. Labor income follows an exogenous process estimated on the Panel Study of Income Dynamics (PSID). I keep the economies that also match the PSID's wealth mobility profile at ten years and in shape. Where the type is both very persistent and wide, type dependence accounts for 41 to 56 percent. Most existing calibrated models combine high, often permanent, persistence with a type width of 0.1 or less. At these persistences and type widths, type dependence accounts for at most 18 percent of top wealth concentration in the fitted model.

This paper relates most closely to two studies. Gaillard and Wangner (2023) find that pure type dependence and pure scale dependence produce almost the same wealth distribution. I show that with i.i.d. labor income the match is exact: every type-dependent economy has a scale-dependent memoryless twin with the same wealth distribution and the same one-period wealth mobility. Benhabib, Bisin and Fernholz (2022) find that, of the models they compare, only one with permanent heterogeneity in the growth rate of wealth matches wealth mobility across generations. In their models earnings are deterministic within each life and follow a Markov chain across generations. Within a working life labor income is persistent, and with persistent labor income a positive floor in wealth mobility no longer identifies a type.

**Related literature** I contribute to two strands of the literature. First, scale dependence is the state dependence of Heckman (1981), and type dependence is his unobserved heterogeneity. Shorrocks (1976) shows that an iterated one-step transition understates long-horizon persistence in income classes. A strongly persistent type produces the same understatement for wealth.

Second, many calibrated models of top wealth concentration match the wealth distribution and target at most one wealth mobility matrix (Krusell and Smith, 1998; Quadrini, 2000; Cagetti and De Nardi, 2006; Benhabib, Bisin and Zhu, 2011; Cao and Luo, 2017; Benhabib, Bisin and Luo, 2019; Hubmer, Krusell and Smith, 2021; Gaillard and Wangner, 2023; Gaillard et al., 2024). By the first result, these targets contain no information about the persistence of the type. Hubmer, Krusell and Smith (2021) fix  $\rho$  a priori, Benhabib, Bisin and Luo (2019) estimate the persistence of returns across generations at 0.10, with a standard error of 0.49 and returns fixed within each life, and Gaillard and Wangner (2023) and Gaillard et al. (2024) take it from households' flows in and out of equity investment and entrepreneurship in the PSID.

**Roadmap** Section 2 defines type dependence, scale dependence and the parametric random growth model. Section 3 proves the one-step equivalence and states the wealth mobility profile results with i.i.d. and with persistent labor income. Section 4 fits the model to U.S. wealth concentration and tests the economies against ten-year wealth mobility. Section 5 concludes.

## 2 Random growth models of wealth

### 2.1 Type dependence and scale dependence

A random growth model of wealth has the law of motion  $w_{i,t+1} = a_{it} w_{it} + \varepsilon_{i,t+1}$ . The wealth growth factor  $a_{it} > 0$  multiplies a household's wealth from one period to the next, and the innovation  $\varepsilon_{i,t+1} > 0$  is labor income.

**Definition 1** (Type-dependent economy). *A type-dependent economy  $\mathcal{E}^T$  sets the growth factor equal to a persistent household type,  $a_{it} = \theta_{it}$ . The type takes  $J$  values  $0 < \theta_1 < \dots < \theta_J$  and follows a Markov chain independent of  $\{\varepsilon\}$ , with stationary distribution  $\pi$  and transition matrix*

$$\Pi = \rho I + (1 - \rho) \mathbf{1} \pi^\top, \quad \rho \in [0, 1], \quad (1)$$

*with  $\mathbf{1}$  the vector of ones, so a household keeps its type with probability  $\rho$  and otherwise redraws it from  $\pi$ , possibly drawing the same type again, giving  $\text{Corr}(\theta_t, \theta_{t+k}) = \rho^k$ . I call  $\rho$  the persistence of the type, and the permanent case is  $\rho = 1$ .*

**Definition 2** (Scale-dependent economy). *A scale-dependent economy  $\mathcal{E}^S$  sets  $a_{it} = G(w_{it}, \eta_{it})$  for some measurable function  $G$  and an i.i.d. draw  $\eta_{it}$  independent of the household's history. Its growth factor has no memory beyond current wealth. With i.i.d. labor income independent of the household's history and of  $\eta_{it}$  (Assumption 1 of Appendix A), its wealth is Markov.*

In Gabaix et al. (2016) scale dependence means that the growth rate of income depends on its level, and in Gaillard and Wangner (2023) that the return rises with wealth. Here scale dependence means that growth has no memory, whether or not growth rises with wealth.

## 2.2 A parametric random growth model

The model I fit nests both mechanisms in the persistence of the type. Its growth factor multiplies the type, a wealth gradient and a memoryless shock:

$$a_{it} = \theta_{it} g(w_{it}) \eta_{it}. \quad (2)$$

The wealth gradient  $g$  rises with wealth, and its slope  $d \geq 0$  sets how far it rises (Appendix D.1). The memoryless shock  $\eta$  is lognormal with mean one, and its log standard deviation  $\bar{\sigma}$  is the shock dispersion. At  $\rho = 0$  the model is scale-dependent, and at  $\rho = 1$  the type is permanent. Between the two, the relative importance of type dependence and scale dependence depends on  $\rho$  and on the type's width (Definition 3).

Log labor income is the sum of a fixed effect  $z_i$ , a first-order autoregressive component  $u_{it}$  of persistence  $\phi$  and a transitory shock  $e_{it}$ ,

$$\log \varepsilon_{it} = \log c + z_i + u_{it} + e_{it}, \quad u_{it} = \phi u_{i,t-1} + \xi_{it}, \quad (3)$$

with  $z_i$ ,  $\xi_{it}$  and  $e_{it}$  normal with standard deviations  $\sigma_z$ ,  $\sigma_\xi$  and  $\sigma_e$ , and  $c$  setting mean labor income to one. Labor income is i.i.d. when  $z_i$  and  $u_{it}$  are absent, and persistent otherwise.

Each period a household exits with probability  $\delta$ . An entrant with zero wealth, a type drawn from  $\pi$  and new draws of the labor income components replaces it:

$$w_{i,t+1} = (1 - x_{i,t+1}) a_{it} w_{it} + \varepsilon_{i,t+1}, \quad (4)$$

with  $x_{i,t+1}$  an independent draw equal to one with probability  $\delta$ . Without exit, a permanent type above one would make the wealth of its households grow without limit on average. With  $\rho < 1$  the mean-growth condition involves the type chain and the growth factor jointly. Exit keeps the wealth distribution stationary even then, because every household leaves with probability  $\delta$  each period (Remark A.4).

### 3 Identification of the type's persistence

With i.i.d. labor income, wealth inequality and one-period wealth mobility cannot distinguish type dependence from scale dependence, but wealth at three consecutive dates distinguishes any persistent type from its memoryless twin. The long-run limit of the wealth mobility profile distinguishes a permanent type. With persistent labor income, a scale-dependent economy with i.i.d. growth factors has a wealth mobility profile of the same shape as that of a permanent type.

#### 3.1 One-step equivalence

Every type-dependent economy has a memoryless twin, its Markovianization: the Markov chain in wealth alone whose kernel is the one-step conditional law  $P^S(w, \cdot) = \Pr(w_{t+1} \in \cdot \mid w_t = w)$  at the stationary distribution (Definition 6, Appendix A). The memoryless twin gives each household, each period, the growth factor of a randomly chosen household with the same wealth. With i.i.d. labor income the memoryless twin is scale-dependent (Proposition A.1).

**Theorem 3.1** (One-step equivalence). *Take any stationary hidden-state Markov process of Definition 6, with stationary wealth distribution  $F$ . Initialized at  $F$ , its Markovianization has the same stationary wealth distribution and the same stationary joint distribution of  $(w_t, w_{t+1})$  as the process itself. In particular, a type-dependent economy  $\mathcal{E}^T$  and its memoryless twin  $\mathcal{E}^S$  agree on two sets of statistics.*

- (i) *Every wealth inequality statistic: any functional of  $F$ , including the variance, the Gini coefficient and top wealth shares.*
- (ii) *Every one-period wealth mobility statistic: any functional of the joint law of  $(w_t, w_{t+1})$ , including the one-period correlation of wealth and the one-period wealth rank-rank coefficient.*

*Proof.* Appendix A. □

**Corollary 3.2** (Wealth inequality and one-period wealth mobility do not identify  $\rho$  against the memoryless twin). *Fix any persistence of the type  $\rho > 0$ . The memoryless twin matches the stationary wealth distribution and the one-step joint law of the type-dependent economy. So no functional of the cross-section or of a single transition identifies  $\rho$  in any model that contains the memoryless twin.*

*Proof.* Immediate from Theorem 3.1, which holds for a type-dependent economy with any  $\rho$ . □

### 3.2 Identification with i.i.d. labor income

Within the linear two-type class, with two type values and  $a_{it} = \theta_{it}$ , the wealth mobility profile identifies  $\rho$  locally at almost every parameter value (Appendix B.7). The wealth mobility profile is the sequence  $\beta_k = \text{Cov}(w_t, w_{t+k}) / \text{Var}(w_t)$  over horizons  $k \geq 1$ , and in rank form the sequence of wealth rank-rank coefficients.

**Proposition 3.3** (A persistent type against its memoryless twin). *Let  $\mathcal{E}^T$  be a type-dependent economy under Assumptions 1 and 2 of Appendix A, with at least two types of positive stationary probability, and  $\mathcal{E}^S$  its memoryless twin.*

- (i) *Wealth is Markov if and only if  $\rho = 0$ . For  $\rho > 0$ , on a set of positive probability,  $w_{t+2}$  is not independent of  $w_t$  given  $w_{t+1}$ . The two economies then share the law of  $(w_t, w_{t+1})$  and differ in the law of  $(w_t, w_{t+1}, w_{t+2})$ .*
- (ii) *With a permanent type,  $\rho = 1$ , the wealth mobility profile of  $\mathcal{E}^T$  converges to  $V_B/V > 0$ , the between-type share of the stationary wealth variance. The wealth mobility profile of  $\mathcal{E}^S$ , and of every scale-dependent economy with the same stationary distribution under the same assumptions, converges to zero.*

*Proof.* Appendices A and B. □

Part (i) holds because wealth one period ago holds information about the type that wealth today does not.

### 3.3 Persistent labor income as a type

Persistent labor income reproduces the shape of a permanent type's wealth mobility profile, a positive floor plus geometric terms at two rates below one, without any type. I compare a scale-dependent economy with persistent labor income to an economy with two permanent types and i.i.d. labor income. The scale-dependent economy's labor income has a permanent and an autoregressive component.

**Proposition 3.4** (Persistent labor income without a type). *Let an economy have i.i.d. growth factors, with mean  $\mu_a$  and  $\mathbb{E}[a^2] < 1$ . Let labor income be additive in levels,  $\varepsilon_{it} = \bar{\varepsilon} + z_i + u_{it}$ , with bounded components that keep  $\varepsilon_{it} > 0$ . The permanent component  $z_i$  has mean zero and variance  $v_z > 0$ , and  $u_{it} = \phi u_{i,t-1} + \xi_{it}$  with  $\phi \in (0, 1)$ ,  $\phi \neq \mu_a$  and  $\xi$  i.i.d. with mean zero and positive, finite variance. The growth factors,  $z_i$  and the  $\xi_{it}$  are mutually independent, and wealth and  $u$  are at their stationary laws. Then for every  $k \geq 1$  and some constants  $c_a$  and  $c_\phi$  the wealth autocovariance*

$C(k) = \text{Cov}(w_t, w_{t+k})$  satisfies

$$C(k) = c_a \mu_a^k + c_\phi \phi^k + c_0, \quad c_0 = \frac{v_z}{(1 - \mu_a)^2} > 0. \quad (5)$$

The wealth autocovariance is a sum of three geometric terms with rates  $\mu_a$ ,  $\phi$  and one. When  $\mu_a < \phi$ , the three rates play the roles of  $\theta_L$ ,  $\theta_H$  and one in equation (18) (Appendix B.7) for a type-dependent economy with two permanent types, although  $c_a$ , unlike the loading on  $\theta_L^k$  there, can be negative. The wealth mobility profile of this economy converges to  $c_0/\text{Var}(w) > 0$ .

*Proof.* Let  $\mathcal{F}_t$  hold  $w_t, z_i$  and  $u_{it}$ . Each growth factor  $a_{t+j}, j \geq 0$ , is independent of  $\mathcal{F}_t$ , so  $\mathbb{E}[w_{t+k} | \mathcal{F}_t] = \mu_a^k w_t + \sum_{j=1}^k \mu_a^{k-j} \mathbb{E}[\varepsilon_{t+j} | \mathcal{F}_t]$ , with  $\mathbb{E}[\varepsilon_{t+j} | \mathcal{F}_t] = \bar{\varepsilon} + z_i + \phi^j u_{it}$ . Hence

$$C(k) = \mu_a^k \text{Var}(w_t) + \frac{1 - \mu_a^k}{1 - \mu_a} \text{Cov}(w_t, z_i) + \frac{\phi(\mu_a^k - \phi^k)}{\mu_a - \phi} \text{Cov}(w_t, u_{it}).$$

Writing  $w_t$  as the sum of past incomes times products of independent growth factors gives  $\text{Cov}(w_t, z_i) = v_z / (1 - \mu_a)$ . Collecting the terms in  $\mu_a^k, \phi^k$  and one gives equation (5). The loading on  $\phi^k$  is  $\phi \text{Cov}(w_t, u_{it}) / (\phi - \mu_a)$  and has the sign of  $\phi - \mu_a$ , because the same sum gives  $\text{Cov}(w_t, u_{it}) = \sum_{j \geq 0} \mu_a^j \phi^j \text{Var}(u) = \text{Var}(u) / (1 - \mu_a \phi) > 0$ .  $\square$

The proof signs  $c_\phi$ , which has the sign of  $\phi - \mu_a$ , and  $c_0$  is positive, but  $c_a$  can take either sign. The constant term  $c_0$  is the variance of  $z_i / (1 - \mu_a)$ , the permanent-income component of expected wealth. In the two-type economy the corresponding constant term is the between-type variance  $V_B$ . Both constant terms keep the long-run limit of the wealth mobility profile positive, so that limit does not identify a type. Equation (3) is additive in logs rather than in levels, so the proposition illustrates the fitted model's labor income process rather than covering it.

## 4 A fitted random growth model

I fit the random growth model to the U.S. top 10 wealth share. Table 1 lists its parameters and their targets. I call an economy mobility-consistent when it also matches the PSID's wealth mobility profile at ten years and in shape. In every mobility-consistent economy scale dependence accounts for the larger share of top wealth concentration, unless the type is both very persistent and wide.

**Table 1: The parameters of the fitted random growth model**

| Parameter                            | Value              | Target                               |
|--------------------------------------|--------------------|--------------------------------------|
| <i>A. Fixed across economies</i>     |                    |                                      |
| The labor income process             |                    | PSID income mobility                 |
| persistence, $\phi$                  | 0.78 (annual 0.88) |                                      |
| innovation s.d., $\sigma_{\xi}$      | 0.18               |                                      |
| fixed-effect s.d., $\sigma_z$        | 0.70               |                                      |
| transitory s.d., $\sigma_e$          | 0.25               | Güvenen (2009)                       |
| The shock dispersion, $\bar{\sigma}$ | 0.65               | PSID growth factor's dispersion      |
| The type's average, $\bar{\theta}$   | 0.58               | top 10 wealth share, wealth mobility |
| <i>B. Varied across economies</i>    |                    |                                      |
| The type's persistence, $\rho$       | 0 to 1             | —                                    |
| The type's width, $\Delta$           | 0.1 to 0.9         | —                                    |
| The wealth gradient's slope, $d$     | per economy        | top 10 wealth share, 0.74            |

**Notes:** The model period is two years. The labor income standard deviations (s.d.) are annual. Over the two-year period the autoregressive innovation has an s.d. of 0.24. An economy is one pair  $(\rho, \Delta)$ , and the economy without a type has  $\Delta = 0$ . Sections 4.1 to 4.3 give each target.

#### 4.1 The labor income process

I fit the labor income process of equation (3) to the PSID's within-cohort income rank-rank coefficients at two to ten years. I fit three annual parameters. The autoregressive component has a persistence of 0.88 and an innovation standard deviation of 0.18. The fixed effect has a standard deviation of 0.70. The transitory standard deviation is held at 0.25, the value Güvenen (2009) estimates on the PSID. Over the model's two-year period the autoregressive component's persistence is  $\phi = 0.78$ . The fitted process's five income rank-rank coefficients lie within the PSID's sampling error (Appendix D). The labor income process and its calibration are the same in every economy below.

#### 4.2 The shock dispersion

I set the shock dispersion to  $\bar{\sigma} = 0.65$  so that wealth growth differs as much across households in the model as in the PSID.

The memoryless shock is drawn afresh each period, so a household with a high draw this period is no more likely to get one next period, and its dispersion governs how much wealth growth differs across households. In the PSID, the log of a household's wealth growth over two years, leaving out new saving, has a standard deviation of 0.94. Reporting error and the rising wealth gradient also spread measured wealth growth, so the shock dispersion lies below 0.94. With both included, a shock dispersion of 0.65 gives the economy without a type the PSID's

0.94 (Appendix D.2). I use the same value in every economy, and Section 4.4 reports other values.

### 4.3 Economies and the share of type dependence

**Definition 3** (Economy  $\mathcal{E}(\rho, \Delta)$ ). *In the parametric random growth model of equation (2) in Section 2.2, the type takes three values around its average  $\bar{\theta}$ , namely  $\bar{\theta} - \Delta/2$ ,  $\bar{\theta}$  and  $\bar{\theta} + \Delta/2$ , with equal probability, where  $\Delta$  is the type's width. Each period a household keeps its type with probability  $\rho$  and otherwise redraws it from the three values. The economy without a type is  $\Delta = 0$ .*

I consider 118 economies: one without a type, and every pair of thirteen persistences  $\rho$  from 0 to 1 and nine type widths  $\Delta$  from 0.1 to 0.9 (Figure 1). Every economy but two matches the top 10 wealth share of 0.74 in the 2022 Survey of Consumer Finances (Board of Governors of the Federal Reserve System, 2023) through its own fitted slope  $d$  of the wealth gradient, which sets how much faster richer households' wealth grows. Those two, of type width 0.9 and persistence 0.99 or 1, cannot fit it, because the type alone exceeds that share. Every economy also has the same type's average,  $\bar{\theta} = 0.58$ . Apart from the slope, the economies therefore differ only in the type's persistence and width, and wealth mobility rules some of them out. The value 0.58 best fits the top 10 wealth share and the PSID's ten-year wealth rank-rank coefficient in the economy without a type.

**Definition 4** (Shares of type dependence and scale dependence). *Five channels can be switched off in an economy. They are the type's dispersion (every type set to  $\bar{\theta}$ ), the type's persistence ( $\rho = 0$ ), the wealth gradient ( $d = 0$ ), the memoryless shock ( $\bar{\sigma} = 0$ ) and persistent labor income (fixed effect and autoregressive component removed). With all five off, the top 10 wealth share falls from the matched 0.74 to 0.135 in every economy. I call that fall, about 0.6, top wealth concentration. The share of type dependence is the fall in the top 10 wealth share from setting  $\rho = 0$  alone, in percent of top wealth concentration. The share of persistent labor income is the fall from removing the fixed effect and the autoregressive component alone, in percent of top wealth concentration. The share of scale dependence is the remainder.*

The share of type dependence counts only what the type's persistence adds. At  $\rho = 0$  each household's type is redrawn every period, so a household that grows fast this period because of its type is no more likely to grow fast next period. The type then works like the memoryless shock, and its effect counts as scale dependence. A Shapley decomposition that also counts the type's dispersion as type dependence moves the share of type dependence by up to 11 points across the mobility-consistent economies, and Online Appendix Table S3 shows three of them.

**A. Mobility-consistent economies**

Heatmap showing the share of mobility-consistent economies. The y-axis is 'Type width,  $\Delta$ ' (0.1 to 0.9) and the x-axis is 'Persistence of the type,  $\rho$ ' (0 to 1). The color scale indicates: Mobility-consistent, scale dependence larger (dark gray), Mobility-consistent, type dependence larger (black), and Not mobility-consistent (white).

**B. Share of type dependence, percent**

Heatmap showing the share of type dependence, percent. The y-axis is 'Type width,  $\Delta$ ' (0.1 to 0.9) and the x-axis is 'Persistence of the type,  $\rho$ ' (0 to 1). The color scale indicates: Mobility-consistent, scale dependence larger (dark gray), Mobility-consistent, type dependence larger (black), and Type alone exceeds the U.S. top 10 share (light gray).

**C. Share of scale dependence, percent**

Heatmap showing the share of scale dependence, percent. The y-axis is 'Type width,  $\Delta$ ' (0.1 to 0.9) and the x-axis is 'Persistence of the type,  $\rho$ ' (0 to 1). The color scale indicates: Mobility-consistent, scale dependence larger (dark gray), Mobility-consistent, type dependence larger (black), and Type alone exceeds the U.S. top 10 share (light gray).

**D. Share of persistent labor income, percent**

Heatmap showing the share of persistent labor income, percent. The y-axis is 'Type width,  $\Delta$ ' (0.1 to 0.9) and the x-axis is 'Persistence of the type,  $\rho$ ' (0 to 1). The color scale indicates: Mobility-consistent, scale dependence larger (dark gray), Mobility-consistent, type dependence larger (black), and Type alone exceeds the U.S. top 10 share (light gray).

Type dependence then leads in fewer economies: no longer at a type width of 0.3, nor at a type width of 0.4 with a persistence below 0.97.

**Definition 5** (Mobility-consistent economy). *An economy is mobility-consistent if it matches the PSID’s wealth mobility (Van Langenhove, 2026b) at ten years and in shape. First, its within-cohort wealth rank-rank coefficient ten years apart,  $r_{10}$ , lies in  $[0.662, 0.723]$  (Online Appendix Table S2).*

*Second, a Wald test at the 95 percent level does not reject that its differences  $r_k - r_2$  at four, six, eight and ten years equal the PSID's. The test's covariance adds the simulation's to the PSID's.*

I obtain two results. First, the PSID's wealth mobility rules out a wide type unless it is nearly permanent. Every type of width 0.3 or less is mobility-consistent at every persistence, as is the economy without a type (Figure 1, panel A). A wider type is mobility-consistent only at a persistence of 0.95 or more, except at a type width of 0.4 with a persistence of 0.3 or less (Online Appendix S5.2). At a low persistence a wide type is an extra memoryless shock that reshuffles wealth ranks. At a higher persistence it also creates much of top wealth concentration itself, so the fitted wealth gradient is flatter. Either way the wealthy lose their rank faster than in the PSID, unless a lasting type keeps them on top.

Second, scale dependence accounts for the larger share of top wealth concentration in every mobility-consistent economy except where the type is both very persistent and wide (Figure 1, panel A). Type dependence accounts for the larger share only at a persistence of 0.95 or more with a type width of 0.4 or more, and at a persistence of 0.99 or more with a type width of 0.3. In these economies it accounts for 41 to 56 percent. At a type width of 0.3 the two shares nearly tie, and their order holds on new simulation draws (Appendix D.1).

**Robustness** I vary the shock dispersion and the labor income process.

At other values of the shock dispersion, type dependence accounts for the larger share only in economies where wealth growth differs more across households than in the PSID. At  $\bar{\sigma} = 0.55$ , near the smallest value the PSID's growth-factor covariances allow, type dependence accounts for at most 41 percent. At  $\bar{\sigma} = 0.75$ , the value of the PSID growth factor's dispersion net of reporting error, it accounts for the larger share only at a persistence of 0.8 or more. At  $\bar{\sigma} = 0.85$ , the highest estimate of the growth factor's dispersion net of error, it does so at every mobility-consistent permanent type, at every mobility-consistent type of width 0.2 or more and persistence 0.71 to 0.95, and at the type of width 0.4 with a persistence of 0.5. At  $\bar{\sigma} = 0.75$  and 0.85 it accounts for at most 56 percent, and every mobility-consistent economy has wealth growth that differs more across households than in the PSID.

With other labor income processes, type dependence accounts for the larger share only in economies that do not match the PSID. With the more persistent process of Guvenen (2009) and  $\bar{\sigma}$  refit to 0.69, type dependence accounts for the larger share only at the permanent types of width 0.5 or more and one type with a persistence of 0.99, and for at most 49 percent. None of them matches the PSID's wealth growth (Appendix D.3, Online Appendix S5.3). With the labor income processes of Gaillard et al. (2024) and Gaillard and Wangner (2023), which have

**Table 2: The persistent types of existing calibrated models**

| Model                                       | Type            | Persistence, $\rho$ | The type's width, $\Delta$ |
|---------------------------------------------|-----------------|---------------------|----------------------------|
| <i>A. Impermanent types</i>                 |                 |                     |                            |
| Krusell and Smith (1998)                    | discount factor | 0.96                | 0.02                       |
| Krusell et al. (2009)                       | discount factor | 0.96                | 0.02                       |
| Cao and Luo (2017)                          | return          | 0.82                | 0.07                       |
| Bachmann et al. (2020)                      | discount factor | 0.96                | 0.01                       |
| Hubmer, Krusell and Smith (2021)            | discount factor | 0.98                | 0.02                       |
| Atkeson and Irie (2022)                     | wealth growth   | 0.87                | 0.10                       |
| Guvenen et al. (2023)                       | return          | 0.94                | at most 0.12               |
| Song (2025)                                 | return          | 0.92                | 0.03                       |
| Kaymak, Leung and Poschke (2026)            | return          | 0.96                | 0.09                       |
| <i>B. Types permanent over a life</i>       |                 |                     |                            |
| Erosa and Koreshkova (2007)                 | discount factor | 1                   | 0.02                       |
| Hendricks (2007)                            | discount factor | 1                   | 0.06 to 0.12               |
| Benhabib, Bisin and Zhu (2011)              | return          | 1                   | 0.06 to 0.09               |
| Carroll, Slacalek and Tokuoka (2014)        | discount factor | 1                   | 0.02                       |
| Suen (2014)                                 | discount factor | 1                   | 0.02                       |
| Brinca et al. (2016)                        | discount factor | 1                   | 0.06                       |
| Krueger, Mitman and Perri (2016)            | discount factor | 1                   | 0.04                       |
| Carroll et al. (2017)                       | discount factor | 1                   | 0.04 to 0.12               |
| Benhabib, Bisin and Luo (2019)              | return          | 1                   | 0.08                       |
| Brinca et al. (2021)                        | discount factor | 1                   | 0.06                       |
| Xavier (2021)                               | return          | 1                   | 0.03                       |
| Benhabib, Bisin and Fernholz (2022)         | wealth growth   | 1                   | at most 0.03               |
| Ernst et al. (2024)                         | discount factor | 1                   | 0.03                       |
| Gomez and Gouin-Bonenfant (2024)            | discount factor | 1                   | 0.01                       |
| Mahler and Yum (2024)                       | discount factor | 1                   | 0.03                       |
| De Nardi, Pashchenko and Porapakkarm (2025) | discount factor | 1                   | 0.14                       |
| Albers, Kersting and Stieglitz (2026)       | return          | 1                   | 0.03 to 0.05               |
| Catherine, Lu and Paron (2026)              | discount factor | 1                   | 0.04                       |
| Hubmer et al. (2026)                        | return          | 1                   | 0.14                       |
| Hubmer et al. (2026)                        | discount factor | 1                   | 0.32                       |

**Notes:** Each model's persistent type on the two axes of Figure 1: its persistence per two-year period and its type width (Definition 3). A type permanent over a life has  $\rho = 1$ . A range spans the interpretations of the source, and "at most" an upper bound. Online Appendix Table S5 gives each paper's persistence and spread, and Online Appendix S5.4 the conversion and the models left out.

no fixed effect, wealth ranks move too fast. The ten-year wealth rank-rank coefficient is at most 0.54, and no economy is mobility-consistent. Online Appendix S5.3 and Online Appendix Table S4 give the details.

## 4.5 Calibrated models

Table 2 places the persistent types of existing calibrated models on the two axes of Figure 1. Most combine high, often permanent, persistence with a type width of 0.1 or less, where scale

dependence outweighs type dependence.<sup>1</sup> The type is a return, a discount factor or the growth rate of wealth. The models fall into two groups.

First, some models make the type impermanent, with a two-year persistence of 0.82 to 0.98 (Krusell and Smith, 1998; Krusell et al., 2009; Cao and Luo, 2017; Bachmann et al., 2020; Hubmer, Krusell and Smith, 2021; Atkeson and Irie, 2022; Guvenen et al., 2023; Song, 2025; Kaymak, Leung and Poschke, 2026). In Guvenen et al. (2023) the impermanent type is a fast-lane state of entrepreneurial productivity on top of a permanent ability. Their type width is 0.1 or less, except for an upper bound of 0.12 in Guvenen et al. (2023). At a type width of 0.1 and a persistence of 0.98 or less, type dependence accounts for at most 16 percent and scale dependence for at least 51 percent (Figure 1, panels B and C).

Second, most models make the type permanent over a life (Erosa and Koreshkova, 2007; Hendricks, 2007; Benhabib, Bisin and Zhu, 2011; Carroll, Slacalek and Tokuoka, 2014; Suen, 2014; Brinca et al., 2016; Krueger, Mitman and Perri, 2016; Carroll et al., 2017; Benhabib, Bisin and Luo, 2019; Brinca et al., 2021; Xavier, 2021; Benhabib, Bisin and Fernholz, 2022; Ernst et al., 2024; Gomez and Gouin-Bonenfant, 2024; Mahler and Yum, 2024; De Nardi, Pashchenko and Porapakkarm, 2025; Albers, Kersting and Stieglitz, 2026; Catherine, Lu and Paron, 2026; Hubmer et al., 2026). All but one have a type width of 0.14 or less, and every economy with a type width of 0.1 or 0.2 is mobility-consistent (Figure 1, panel A). For a permanent type with a type width of 0.1, scale dependence accounts for 51 percent of top wealth concentration and type dependence for 18 percent (Figure 1, panels B and C).

## 5 Conclusion

A calibration that chooses the persistence of a return or a discount factor thereby chooses the relative importance of type dependence and scale dependence that it reports. Every economy with a persistent type has a memoryless twin with the same wealth distribution and the same one-step transition, so neither a cross-section nor a single transition can settle that persistence. With i.i.d. labor income, wealth over several periods distinguishes a persistent type

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<sup>1</sup>Table 2 leaves out the models whose type is entrepreneurial ability, an investor type or a preference and whose papers report no gap that converts into a type width (Cagetti and De Nardi, 2006; Kitao, 2008; Favilukis, 2013; Peter, 2020; Boar and Midrigan, 2023; Gaillard and Wangner, 2023; Fernández-Villaverde and Levintal, 2024; Gaillard et al., 2024; Rotberg and Steinberg, 2024; Beare and Toda, 2025; Brüggemann and Mahone, 2025; Gomez, 2025; Guo, 2026; Pieroni, 2026; Di Nola, Scholl and Wang, 2026). Two working papers estimate or calibrate wider return states that mean-revert within a few years (Pugh, 2021; Sundram, 2025). Their type widths are 0.26 or more at a two-year persistence below 0.5, where type dependence accounts for at most 13 percent in every mobility-consistent economy with a persistence of 0.5 or less. The discount-factor model of Hubmer et al. (2026) has a type width of 0.32 when its dispersion is read as a standard deviation, close to the type width of 0.3 where the two shares nearly tie. Online Appendix S5.4 gives the search and the conversion.

from its memoryless twin. Persistent labor income, however, gives a scale-dependent economy a wealth mobility profile of the same shape as a permanent type's, and ten years of U.S. wealth mobility are matched both by an economy without a type and by economies with a persistent type.

I fit the random growth model to the top 10 wealth share of the 2022 Survey of Consumer Finances and keep the economies that match the PSID's wealth mobility profile at ten years and in shape. Among them, type dependence accounts for the larger share of top wealth concentration only where the type is both very persistent and wide. That region holds the types with a persistence of 0.95 or more and a type width of 0.4 or more, and those with a persistence of 0.99 or more and a type width of 0.3. In that region type dependence accounts for 41 to 56 percent.

Most existing calibrated models combine a persistent type in the return, the discount factor or the growth rate of wealth with a type width of 0.1 or less. At those persistences and type widths, the fitted model attributes at most 18 percent of top wealth concentration to type dependence and at least 50 percent to scale dependence.

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## A The equivalence

Both economies share one law of motion for wealth. They differ only in what the growth factor depends on: a persistent latent type (type dependence) or current wealth (scale dependence).

### A.1 The wealth accumulation law

Time is discrete,  $t = 0, 1, 2, \dots$ , and one period is one survey wave of two years. Every persistence and decay rate is per period. Households  $i \in [0, 1]$  form a unit mass. Household  $i$  has wealth  $w_{it} \geq 0$ , which evolves as

$$w_{i,t+1} = a_{it} w_{it} + \varepsilon_{i,t+1}. \quad (6)$$

The growth factor  $a_{it} > 0$  multiplies a household's own wealth from one period to the next, and the additive innovation  $\varepsilon_{i,t+1} > 0$  is labor income or other inflows. Equation (6) is the multiplicative law of motion of the random growth literature, here in its mean-reverting regime (Gabaix, 2009; Benhabib and Bisin, 2018).

**Assumption 1** (Innovations). *The innovations  $\varepsilon_{i,t+1}$  are i.i.d. across households and time. Each  $\varepsilon_{i,t+1}$  is independent of the household's history to date  $t$ : its past wealth and innovations, and its current and past growth factors and the draws that form them. The mean is  $\mathbb{E}[\varepsilon] = 1/\lambda$  and the variance is  $\text{Var}(\varepsilon) = \sigma_\varepsilon^2 \in (0, \infty)$ . The innovation has a density that is positive on  $\mathbb{R}_+$  and bounded away from zero on every compact subset of  $(0, \infty)$ . The running example uses the exponential law  $\varepsilon \sim \text{Exp}(\lambda)$  with rate  $\lambda$ , for which  $\sigma_\varepsilon^2 = 1/\lambda^2$ .*

**Assumption 2** (Contraction). *Growth factors satisfy  $a_{it} \leq \bar{a} < 1$  almost surely.*

Under Assumptions 1 and 2, each economy has a stationary distribution with finite variance (Appendix A.5). The distribution is unique for  $\rho < 1$ , and at  $\rho = 1$  it is unique given the stationary type shares of Definition 1. The covariance results use only the first two moments of  $\varepsilon$ . The exponential law serves only the closed forms of Appendix B and the illustrative figures of the Online Appendix.

Under the contraction, stationary wealth is no heavier-tailed than income. With exponential income it therefore has no Pareto tail. The qualitative results do not need the light tail. Without the contraction, the one-step equivalence holds, multi-period wealth mobility in rank form distinguishes a permanent type (Proposition B.4), and the three-period law distinguishes a persistent two-valued type under a likelihood-ratio condition (Online Appendix S4.2). The results stated in levels, through wealth autocovariances, need a finite wealth variance, which

the contraction guarantees. Three results relax an assumption: Proposition 3.4 makes labor income persistent, Remark A.4 replaces the contraction by exit, and Proposition B.4 lets the growth factor exceed one.

## A.2 Type-dependent economy

The type follows the chain of Definition 1, equation (1): each period a household keeps its type with probability  $\rho$  and otherwise redraws it from the stationary distribution  $\pi$ .  $\rho = 1$  gives permanent types,  $\rho = 0$  a type redrawn every period, and values in between mean reversion. A type of zero stationary probability is never drawn, so I drop it and take  $\pi_j > 0$  for every  $j$ . With two types I write  $\theta_L = \theta_1$ ,  $\theta_H = \theta_2$  and  $\pi = (1 - p, p)$ . The mean type is  $\bar{\theta} = \mathbb{E}[\theta]$ .

A redraw forgets the current type. For any function  $f$  of the type, the expected deviation of  $f(\theta_{t+1})$  from its mean is therefore  $\rho$  times the current deviation:  $\mathbb{E}[f(\theta_{t+1}) \mid \theta_t] - \mathbb{E}[f(\theta)] = \rho (f(\theta_t) - \mathbb{E}[f(\theta)])$ . Iterating over  $k$  periods gives, in the stationary distribution and for any number of types  $J$ ,

$$\text{Cov}(f(\theta_t), f(\theta_{t+k})) = \rho^k \text{Var}(f(\theta_t)). \quad (7)$$

With  $f(\theta) = \theta$  it gives  $\text{Corr}(\theta_t, \theta_{t+k}) = \rho^k$ .

Under equation (1), a household that loses its type redraws it from  $\pi$  and keeps nothing of the old one, so every covariance decays at the single rate  $\rho$ . A general chain on  $J$  types can decay at up to  $J - 1$  rates, its eigenvalues other than one. If the chain is diagonalizable, the covariance in equation (7) becomes a weighted sum of the  $k$ -th powers of these eigenvalues. The single rate enters the proof of part (i) of Proposition 3.3 and the recovery of  $\rho$  in Appendix B. The other qualitative results need only that a persistent type makes wealth non-Markov. Two types are the running case. Online Appendix S2.3 shows that the number of types is identified when the rates are all different and their loadings nonzero.

In  $\mathcal{E}^T$  wealth is informative about the growth factor  $a_{it} = \theta_{it}$  but does not determine it. This uncertainty alone does not break the Markov property: at  $\rho = 0$  the factor is dispersed given wealth, yet wealth is Markov. For  $\rho > 0$  wealth is not a sufficient statistic, because earlier wealth changes the type's posterior given current wealth (Proposition 3.3, part (i)). The pair  $(w_{it}, \theta_{it})$  is Markov, and wealth alone is a hidden Markov process.

## A.3 The Markovianization

The Markovianization is a scale-dependent economy that reproduces a type-dependent economy's wealth distribution and its entire one-step transition.

**Definition 6** (Markovianization). *Let  $(w_t, h_t)$  be a stationary Markov process with  $w_t \in \mathbb{R}_+$ , a hidden state  $h_t$  on a Polish space, and stationary wealth marginal  $F$ . The Markovianization is the Markov chain in wealth alone with transition kernel:*

$$P^S(w, \cdot) \equiv \Pr(w_{t+1} \in \cdot \mid w_t = w), \quad (8)$$

*the conditional law of next wealth given current wealth, evaluated at the stationary distribution.*

Since wealth takes values in  $\mathbb{R}_+$ , this conditional law exists, and it is unique except on a set of wealth levels of probability zero under  $F$ . For a type-dependent economy  $\mathcal{E}^T$  the hidden state is the type, and I write  $\mathcal{E}^S$  for its Markovianization. Let  $\varpi(w) \equiv \Pr^T(\theta_t = \theta_H \mid w_t = w)$  be the probability of the high type given wealth  $w$  in  $\mathcal{E}^T$ , well defined because the joint law has a density under Assumption 1. The Markovianization draws  $\theta_H$  with probability  $\varpi(w)$  and  $\theta_L$  otherwise, then sets  $w_{t+1} = \theta w + \varepsilon_{t+1}$ .<sup>2</sup>

**Proposition A.1** ( $\mathcal{E}^S$  is a scale-dependent economy). *Under Assumption 1, the Markovianization of a type-dependent economy  $\mathcal{E}^T$  (equation (8)) is a scale-dependent economy in the sense of Definition 2.*

Two households with the same wealth can draw different growth factors in  $\mathcal{E}^S$ , yet  $\mathcal{E}^S$  is not type-dependent: each draw depends only on current wealth and a fresh random number.

**Remark A.2 (The Markovianization can have a rising wealth gradient).** Because  $P^S(w, \cdot)$  is the type-dependent economy's own one-step law, the Markovianization reproduces the whole conditional distribution of next wealth. When the probability of the high type,  $\varpi(w)$ , rises with wealth, the expected growth factor rises with wealth and its conditional variance stays positive. The Markovianization is then a scale-dependent economy with a rising wealth gradient.

#### A.4 The scope of the equivalence

Theorem 3.1 holds for any saving rule, with the exit and entry of the fitted model, and with an observable age (Remarks A.3 to A.5). Remark A.6 compares the theorem with the non-identifiability of a mixture. Appendix A.5 proves the theorem.

**Remark A.3 (Any saving rule).** Theorem 3.1 does not use the linear law of equation (6). The Markovianization redraws the hidden state from its distribution given current wealth, whatever rule maps the state into next wealth. So the equivalence holds for a nonlinear saving rule, such as the policy function of the income-fluctuation model (Online Appendix S1).

<sup>2</sup>The construction is the discrete-time analogue of a mimicking Markov chain (Gyöngy, 1986): a Markov process matched to the one-step conditional laws of a richer, non-Markov process.

**Remark A.4 (Exit and entry).** The fitted model of equation (4) adds exit at rate  $\delta$  and entry at zero wealth, with a type drawn from  $\pi$ . Theorem 3.1 still holds, because exit is independent of wealth and type and the Markovianization's kernel includes it. Exit also replaces the contraction of Assumption 2. Each period a household restarts from income with probability  $\delta$ , so  $P^S(w, \cdot) \geq \delta \Pr(\varepsilon \in \cdot)$  for every  $w$ . Likewise, with probability  $\delta$  the joint chain of wealth, type and labor income components moves to an entrant's next state, whose law does not depend on the current state. This joint chain is uniformly ergodic at every type persistence and type level (Meyn and Tweedie, 2009, Theorem 16.0.2), so the type may exceed one. Stationary wealth may then have infinite variance, but the wealth rank-rank coefficient does not need a finite one.

With i.i.d. labor income and the factor  $a_{it} = \theta_{it}$ , part (i) of Proposition 3.3 holds with exit for households present at all three dates, because given survival the type is as persistent as without exit. The memoryless comparison is then the Markovianization on wealth and age (Remark A.5). With the factor  $\theta g(w)\eta$  of equation (2), the posterior argument in the proof of part (i) does not apply as written, and the limits of part (ii) concern horizons beyond a working life. So I simulate the wealth mobility profiles of the fitting economies (Sections 4.3 and 4.4), as within-cohort wealth rank-rank coefficients of the households present at both dates.

**Remark A.5 (Observable age).** With exit and entry at zero wealth, a household's age since entry,  $\tau_t$ , is observable: it rises by one each period and returns to zero at exit. Definition 6 and Theorem 3.1 hold with the state  $(w_t, \tau_t)$  in place of  $w_t$ , with the same proof, which uses only the stationarity of the joint process. The hidden state is the type under i.i.d. labor income, and the type with the labor income components  $(z_i, u_{it})$  under the labor income process of Section 4.1. This Markovianization shares with  $\mathcal{E}^T$  the stationary joint law of wealth and age and the one-step joint law, so it matches every within-cohort statistic over one transition. Its kernel depends on age, so it is not a scale-dependent economy, and Proposition A.1 does not apply to it.

**Remark A.6 (More than a mixture).** The equivalence is sharper than the non-identifiability of a mixture of Markov chains, because the Markovianization is itself a scale-dependent economy (Proposition A.1 and Remark A.2). Under rank conditions, static mixtures are identified from three observations (Hall and Zhou, 2003; Allman, Matias and Rhodes, 2009; Bonhomme, Jochmans and Robin, 2016) and dynamic mixtures from six (Kasahara and Shimotsu, 2009). The positive identification of Appendix B follows that line. The equivalence matters because the calibrated models of Section 1 match the stationary distribution and at most one wealth mobility matrix, and in any model class that contains the memoryless twin these leave  $\rho$  free. The two economies first differ in the three-period law (Proposition 3.3).

## A.5 Stationary distributions and proofs

Every economy has a stationary distribution. For  $\rho < 1$  its wealth autocovariance vanishes at a geometric rate, and at  $\rho = 1$  it does not. The proofs of Proposition A.1, Theorem 3.1 and part (i) of Proposition 3.3 follow.

Take  $w_{t+1} = a_t w_t + \varepsilon_{t+1}$  with  $a_t \leq \bar{a} < 1$ , and an i.i.d. innovation  $\varepsilon_{t+1} > 0$  with  $\mathbb{E}\varepsilon^2 < \infty$  and a density positive on  $\mathbb{R}_+$  (Assumptions 1 and 2). This covers each fixed-coefficient process, the scale-dependent economies with  $a_t = G(w_t, \eta_t)$ , and, for  $\rho < 1$ , the type-dependent economy's joint chain  $(w_t, \theta_t)$ .

**Existence and uniqueness.** For a fixed type, wealth is a contractive random affine recursion, with a unique stationary distribution reached from any start (Diaconis and Freedman, 1999) and the moments in equation (10). For the scale-dependent and joint chains, take the Lyapunov function  $W(w) = 1 + w^2$ . Since  $a_t \leq \bar{a} < 1$ , and  $\varepsilon_{t+1}$  has finite variance and is independent of  $(a_t, w_t)$ ,  $\mathbb{E}[w_{t+1}^2 \mid w_t = w] \leq \bar{a}^2 w^2 + 2\bar{a} w \mathbb{E}\varepsilon + \mathbb{E}\varepsilon^2$ . The leading coefficient is  $\bar{a}^2 < 1$ , so  $\mathbb{E}[W(w_{t+1}) \mid w_t = w] \leq \kappa W(w) + L$  with  $\kappa < 1$  and  $L < \infty$ . The innovation density is positive and bounded away from zero on compacts, so the chain is irreducible and aperiodic, and compact sets are small. By the drift criterion (Meyn and Tweedie, 2009, Theorem 15.0.1), the chain is positive Harris recurrent and  $W$ -geometrically ergodic, so the stationary distribution has finite variance.

**Geometric decay of the autocovariance.** By Jensen's inequality the drift inequality for  $W$  gives one for  $\sqrt{W}$ ,

$$\mathbb{E}[\sqrt{W(w_{t+1})} \mid w_t = w] \leq \sqrt{\kappa} \sqrt{W(w)} + \sqrt{L},$$

so the chain is also  $\sqrt{W}$ -geometrically ergodic. This ergodicity gives geometric decay of  $\text{Cov}(\ell(w_t), \ell(w_{t+k}))$  for every  $\ell$  with  $|\ell| \leq \sqrt{W}$ , because  $R\chi^k F(|\ell - F\ell|\sqrt{W}) \leq 2R\chi^k F(W) < \infty$ , with  $F$  the stationary distribution and  $R < \infty, \chi < 1$  the constants of  $\sqrt{W}$ -geometric ergodicity (Meyn and Tweedie, 2009, Chapter 16). With  $\ell(w) = w$ , the wealth autocovariance exists and falls to zero geometrically. So the scale-dependent economy is ergodic, as Propositions B.2 and 3.3 use.

**The type-dependent economy at  $\rho < 1$ .** The joint chain  $(w_t, \theta_t)$  satisfies the same conditions with  $W(w, \theta) = 1 + w^2$ . The inequality  $\theta_t \leq \bar{a}$  holds for every type. All entries of  $\Pi$  are positive, and with the positive innovation density they make compact product sets small. So the joint chain is irreducible, aperiodic and geometrically ergodic. Since  $\ell(w, \theta) = w$  satisfies  $|\ell| \leq \sqrt{W}$ , the type-dependent economy's wealth autocovariance vanishes geometrically at every  $\rho < 1$ .

**Non-ergodicity under permanent types.** At  $\rho = 1$  the type never changes, so the state space splits into one closed class per type. The wealth marginal is a nontrivial mixture of the type-conditional invariant laws: invariant but not ergodic. The wealth autocorrelation tends to  $V_B/V > 0$ , not zero (Lemma B.1).

*Proof of Proposition A.1.* The kernel  $P^S(w, \cdot)$  depends on current wealth alone, so wealth is Markov. Write the factor as  $a_t = G(w_t, \eta_t)$ , with  $\eta_t \sim \text{Unif}(0, 1)$  independent of the history, and set  $G(w, \eta) = \theta_H$  if  $\eta \leq \omega(w)$  and  $\theta_L$  otherwise. This draw reproduces  $P^S$ , because the kernel identity  $P^S(w, A) = \sum_j \Pr(\theta_t = \theta_j \mid w_t = w) \Pr(\theta_j w + \varepsilon \in A)$  needs only that  $\varepsilon_{t+1}$  is independent of  $(w_t, \theta_t)$ . Assumption 1 and the independence of the type chain from  $\{\varepsilon\}$  in Definition 1 give that independence. With  $J$  types,  $\omega$  is the posterior vector. The uniforms  $\eta_t$  are serially independent by construction, and the factor's conditional law depends on current wealth only. The representation does not need the factors to be independent across dates: they can be dependent through wealth.  $\square$

*Proof of Theorem 3.1. Invariance of  $F$ .* For any measurable set  $A$ , integrating the one-step law against  $F$  marginalizes the hidden state:

$$\int P^S(w, A) dF(w) = \int \Pr(w_{t+1} \in A \mid w_t = w) dF(w) = \Pr(w_{t+1} \in A) = F(A).$$

The last equality holds because  $F$  is the type-dependent economy's stationary wealth marginal. So  $F$  is invariant for  $P^S$ , and the Markovianization started at  $F$  is stationary. Under Assumptions 1 and 2 the start does not matter: by the existence result above,  $F$  is the unique stationary distribution of  $\mathcal{E}^S$ , reached from any start.

**Equality of the one-step joint law.** Under  $F$ , the Markovianization's joint law of  $(w_t, w_{t+1})$  is  $dF(w) P^S(w, \cdot)$ . By construction this equals  $dF(w) \Pr(w_{t+1} \in \cdot \mid w_t = w)$ , the type-dependent economy's joint law.

Parts (i) and (ii) follow. A wealth inequality statistic is a functional of  $F$ , and a one-period wealth mobility statistic a functional of the joint law of  $(w_t, w_{t+1})$ . Both agree across the two economies.  $\square$

*Proof of Proposition 3.3, part (i).* Theorem 3.1 gives the shared law of  $(w_t, w_{t+1})$ . For the Markov claim, the conditional law of  $w_{t+2}$  given  $(w_{t+1}, w_t)$  is the mixture of the laws  $\theta_j w_{t+1} + \varepsilon$ ,  $j = 1, \dots, J$ , with weights the posteriors  $\Pr(\theta_{t+1} = \theta_j \mid w_{t+1}, w_t) = (1 - \rho)\pi_j + \rho \Pr(\theta_t = \theta_j \mid w_t, w_{t+1})$ . Take the top type  $\theta_J$  and fix  $w_{t+1} = x > 0$ . The innovation is positive, so  $w_{t+1} > \theta_t w_t$ ,

and the top type is feasible only when  $w_t < x/\theta_j$ . The posterior  $\Pr(\theta_t = \theta_j \mid w_t, w_{t+1})$  is therefore zero for  $w_t > x/\theta_j$  and strictly positive for  $w_t < x/\theta_j$ , by the full-support conditionals and  $\pi_j \in (0, 1)$ . Given  $w_{t+1} = x$ , both regions have positive conditional probability:  $w_t < x/\theta_j$  with a small  $w_t$  and  $\varepsilon$  near  $x$ , and  $w_t \in (x/\theta_j, x/\theta_1)$  with the type  $\theta_1$  and  $\varepsilon = x - \theta_1 w_t > 0$ . For  $\rho > 0$  the weight on  $\theta_j$  then varies with  $w_t$ .

The laws  $\theta_j x + \varepsilon$  are translates of one density by different amounts, and finitely many such translates are linearly independent. So a change in the weights changes the mixture, and the conditional law of  $w_{t+2}$  depends on  $w_t$  on a set of positive probability: wealth is non-Markov. The Markovianization is Markov by construction, so the two economies differ in the law of  $(w_t, w_{t+1}, w_{t+2})$ . At  $\rho = 0$  the type  $\theta_{t+1}$  is drawn from  $\pi$  independently of the past, so  $w$  is Markov. □

## B Propositions on the wealth mobility profile

A type-dependent economy and its memoryless twin share the one-period law, so they first differ in the three-period law (Proposition 3.3). The wealth mobility profile is a linear summary of that difference that panel data can measure.

I prove three results. First, the three-period law distinguishes any persistent type from its memoryless twin (Proposition 3.3, part (i), proved in Appendix A.5). Second, the long-run limit of the wealth mobility profile distinguishes a permanent type (part (ii)): at  $\rho = 1$  wealth is non-ergodic and the gap never closes. Third, within the linear two-type class, the wealth mobility profile identifies  $\rho$  locally (Proposition B.7). There the wealth autocovariance has a component at the rate  $\rho$ , which fades more slowly as  $\rho$  rises. As  $\rho$  falls to zero, the gap closes at every horizon (Online Appendix Proposition S2.1). I do not prove a gap at every horizon for every mean-reverting type against every scale-dependent economy.

### B.1 The wealth mobility profile

The wealth mobility profile is the sequence of wealth autocorrelations  $\beta_k$  over horizons  $k$ . With stationary mean  $\mu = \mathbb{E}[w]$  and variance  $V = \text{Var}(w)$ ,  $\beta_k$  is the slope of  $w_{t+k}$  on  $w_t$ :

$$\beta_k = \frac{\text{Cov}(w_{t+k}, w_t)}{\text{Var}(w_t)} = \frac{C(k)}{V}, \quad C(k) \equiv \text{Cov}(w_{t+k}, w_t). \quad (9)$$

A higher  $\beta_k$  means more persistent wealth and lower wealth mobility, and  $\beta_0 = 1$ .

### B.2 Within-type and between-type wealth variance

With its type held at  $\theta$ , a household's wealth follows the fixed-coefficient process  $w_{t+1} = \theta w_t + \varepsilon_{t+1}$ , whose stationary moments solve  $\mathbb{E}[w] = \theta \mathbb{E}[w] + 1/\lambda$  and  $\text{Var}(w) = \theta^2 \text{Var}(w) + \sigma_\varepsilon^2$ , which gives:

$$\mu_\theta = \frac{1/\lambda}{1 - \theta}, \quad v_\theta = \frac{\sigma_\varepsilon^2}{1 - \theta^2}, \quad \text{Cov}(w_t, w_{t+k} \mid \theta) = \theta^k v_\theta. \quad (10)$$

With permanent types the cross-section mixes these laws with weights  $(1 - p, p)$ , and the law of total variance gives:

$$V = \underbrace{\sum_\theta \pi_\theta v_\theta}_{\text{within type}} + \underbrace{\sum_\theta \pi_\theta (\mu_\theta - \mu)^2}_{\text{between type}}, \quad \pi_{\theta_H} = p, \pi_{\theta_L} = 1 - p, \quad (11)$$

with the within-type variance  $V_W = \sum_{\theta} \pi_{\theta} v_{\theta}$  and the between-type variance  $V_B = \sum_{\theta} \pi_{\theta} (\mu_{\theta} - \mu)^2 = p(1-p)(\mu_H - \mu_L)^2 > 0$ , writing  $\mu_H \equiv \mu_{\theta_H}$  and  $\mu_L \equiv \mu_{\theta_L}$ . The identification results rest on  $V_B$ , the wealth dispersion that permanent differences in the growth factor create.

### B.3 The permanent floor in the autocovariance

**Lemma B.1** (Autocovariance with permanent types). *In the type-dependent economy with permanent types ( $\rho = 1$ ), the stationary wealth autocovariance is, for all  $k \geq 0$ :*

$$C^T(k) = \underbrace{\sum_{\theta} \pi_{\theta} \theta^k v_{\theta}}_{\text{within type, decaying}} + \underbrace{V_B}_{\text{between type, permanent}}. \quad (12)$$

So  $\beta_k^T = C^T(k)/V$  falls from  $\beta_0^T = 1$  to the limit  $V_B/V > 0$ .

The autocovariance has a transitory part, which decays within each type at rate  $\theta$ , and a permanent part,  $V_B$ , the spread of mean wealth across types. Because of the permanent part, the long-run rank transition does not converge to independence: a high-type household stays above a low-type household in expectation.

### B.4 The scale-dependent benchmark

The Markovianization matches every cross-sectional and one-period moment but not, under permanent types, the long-horizon wealth mobility profile, because it has no persistent type and forgets its past. Proposition 3.3 is stated for the coefficient  $\beta_k$  in levels under a finite wealth variance, where the gap has the closed form  $V_B/V$ . The proposition's rank form, which the PSID evidence uses, also holds under a Pareto tail generated by a Kesten law (Proposition B.4).

**Proposition B.2** (The Markovianization is ergodic). *The Markovianization  $\mathcal{E}^S$  of a type-dependent economy that satisfies Assumptions 1 and 2 is ergodic. Its autocovariance satisfies  $C^S(1) = C^T(1)$  and  $C^S(k) \rightarrow 0$ , so  $\beta_k^S \rightarrow 0$ .*

*Proof of Proposition B.2.* The kernel  $P^S(w, \cdot)$  generates  $w' = \theta w + \varepsilon$ . The draw takes  $\theta_H$  with probability  $\omega(w)$  and  $\theta_L$  otherwise, with  $\omega(w) \in (0, 1)$  for every  $w > 0$  because both type-conditional densities are positive on  $(0, \infty)$  (Assumption 1) and  $p \in (0, 1)$ . The innovation has a density positive on  $\mathbb{R}_+$  (Assumption 1). So the chain is irreducible and aperiodic. The contraction  $\bar{a} < 1$  with  $\mathbb{E}\varepsilon^2 < \infty$  gives a geometric drift condition, so the chain is positive Harris recurrent and geometrically ergodic (Appendix A.5). Geometric ergodicity gives  $\text{Cov}(w_{t+k}, w_t) \rightarrow 0$ . The equality  $C^S(1) = C^T(1)$  is part (ii) of Theorem 3.1.  $\square$

Part (ii) of Proposition 3.3 follows. Let  $\mathcal{E}^T$  have permanent types, and let  $\mathcal{E}^S$  be any scale-dependent economy with the same stationary wealth distribution. Under Assumptions 1 and 2,  $\mathcal{E}^S$  is geometrically ergodic, so its wealth autocovariance vanishes at long horizons (Appendix A.5). Then:

$$\lim_{k \rightarrow \infty} \beta_k^T = \frac{V_B}{V} > 0 = \lim_{k \rightarrow \infty} \beta_k^S. \quad (13)$$

So no scale-dependent economy with i.i.d. labor income matches the type-dependent economy's long-run wealth mobility, although the Markovianization matches its wealth inequality and one-period wealth mobility exactly (Theorem 3.1).

The wedge is the between-type variance  $V_B$ . Against the memoryless twin, neither the cross-section nor any single transition distinguishes  $V_B$  from the within-type variance.

**Remark B.3 (Rank-based wealth mobility and heavy tails).** The result also holds for rank-based wealth mobility. Geometric ergodicity makes  $w_t$  and  $w_{t+k}$  independent in the limit. So in the scale-dependent economy every long-horizon wealth mobility measure tends to its non-persistence value: the wealth rank-rank coefficient tends to zero and the rank transition matrix to independence. Under permanent types neither does, because a high type stays above a low type in distribution (the dominance and rank-limit steps of the proof of Proposition B.4 apply with  $\eta \equiv 1$ ). The one-period wealth rank-rank coefficient cannot distinguish the type-dependent economy from its memoryless twin, but its long-horizon limit can.

The rank result does not need the baseline's finite moments and extends to the heavy-tailed wealth laws of Online Appendix S4.1, where  $\beta_k$  is undefined once the variance is infinite but the scale-dependent class of Proposition B.4, which contains the Markovianization, stays ergodic. The results stated in levels, including the recovery of  $\rho$  (Appendix B.7), need a finite wealth variance.

**Proposition B.4** (Heavy-tailed wealth: the result in rank form). *Let the types be permanent,  $\rho = 1$ , and let the growth factor be  $a_{it} = \theta_{it}\eta_{it}$ , with the within-type multiplier  $\eta_{it}$  of Online Appendix S4.1. Replace the contraction (Assumption 2) by a Kesten condition on each type:  $\mathbb{E}[\log(\theta_s\eta)] < 0$  and  $\Pr(\theta_s\eta > 1) > 0$ , and there is  $\zeta_s > 0$  with  $\mathbb{E}[(\theta_s\eta)^{\zeta_s}] = 1$ ,  $\mathbb{E}[(\theta_s\eta)^{\zeta_s} \log^+(\theta_s\eta)] < \infty$ , and  $\mathbb{E}[\varepsilon^{\zeta_s}] < \infty$ , with  $\log(\theta_s\eta)$  non-arithmetic. Each type-conditional stationary wealth law then has a Pareto tail, and the high type has the heavier one. The coefficient  $\beta_k$  may be undefined. Restrict the scale-dependent class to growth factors with  $\sup_w \mathbb{E}[G(w, \eta')^u] \leq \kappa < 1$  for some  $u \in (0, 1]$ , with  $\eta'$  the memoryless draw of Definition 2, a class that contains the Markovianization. Every economy in that class is geometrically ergodic, so its long-horizon rank-rank wealth mobility decays to in-*

dependence. The type-conditional stationary laws are strictly ordered by first-order stochastic dominance. With  $F$  the pooled stationary distribution, the wealth rank-rank coefficient then converges to  $12 \text{Var}_\pi(\mathbb{E}[F(w) \mid \theta]) > 0$ . Part (ii) of Proposition 3.3 holds in rank form. The proof is in Online Appendix S4.2.

**Corollary B.5** (What the gap depends on). *The gap  $V_B/V$  in equation (13) is positive whenever the two types differ and both occur, and approaches one as  $\theta_H \rightarrow 1$  with  $\theta_L$  fixed. Under the exponential law:*

$$V_B = \frac{p(1-p)}{\lambda^2} \left( \frac{1}{1-\theta_H} - \frac{1}{1-\theta_L} \right)^2, \quad V_W = \frac{1}{\lambda^2} \left( \frac{1-p}{1-\theta_L^2} + \frac{p}{1-\theta_H^2} \right), \quad (14)$$

and the ratio  $V_B/V$  does not depend on  $\lambda$ .

### B.5 Autocovariance with persistent types

Let  $h_k(s, w) = \mathbb{E}[w_{t+k} \mid \theta_t = s, w_t = w]$  in the type-dependent economy, with transition  $\Pi$  and  $D = \text{diag}(\theta_L, \theta_H)$ . Then  $h_k$  is affine in  $w$ :

$$h_k(s, w) = \alpha_k(s) + b_k(s) w, \quad (15)$$

$$b_k = (D\Pi)^k \mathbf{1}, \quad \alpha_k = \Pi(\alpha_{k-1} + \frac{1}{\lambda} b_{k-1}), \quad \alpha_0 = 0, \quad b_0 = \mathbf{1}.$$

**Proof by induction on  $k$ .** At  $k = 0$ ,  $h_0(s, w) = w$ , so  $b_0 = \mathbf{1}$  and  $\alpha_0 = 0$ . Assume  $h_{k-1}(s, w) = \alpha_{k-1}(s) + b_{k-1}(s) w$  is affine. Given  $\theta_t = s$  and  $w_t = w$ , one has  $w_{t+1} = \theta_s w + \varepsilon_{t+1}$  and  $\theta_{t+1} \sim \Pi(s, \cdot)$ . By the tower property:

$$\begin{aligned} h_k(s, w) &= \sum_{s'} \Pi_{ss'} \mathbb{E}[h_{k-1}(s', w_{t+1}) \mid \theta_t = s, w_t = w] \\ &= \sum_{s'} \Pi_{ss'} [\alpha_{k-1}(s') + b_{k-1}(s')(\theta_s w + \frac{1}{\lambda})]. \end{aligned}$$

Matching coefficients gives  $b_k(s) = \theta_s \sum_{s'} \Pi_{ss'} b_{k-1}(s')$ , that is  $b_k = D\Pi b_{k-1}$ , and the stated  $\alpha_k$  recursion. So  $b_k = (D\Pi)^k \mathbf{1}$ .

The wealth autocovariance is then:

$$C(k) = \text{Cov}(w_t, h_k(\theta_t, w_t)) = \text{Cov}(w_t, \alpha_k(\theta_t)) + \text{Cov}(w_t, b_k(\theta_t)w_t). \quad (16)$$

The slope  $b_k = (D\Pi)^k \mathbf{1}$  decays at the spectral radius  $\varrho(D\Pi)$ . The intercept  $\alpha_k$  evolves through  $\Pi$ , whose eigenvalues are one and  $\rho$ . The unit eigenvalue gives a component of  $\alpha_k$  that is the same for every type, which drops from  $\text{Cov}(w_t, \alpha_k(\theta_t))$ . For  $\rho < 1$  the slope channel and the intercept channel combine, and  $C(k)$  decays to zero, with  $\limsup_k |C(k)|^{1/k} \leq \max\{\rho, \varrho(D\Pi)\}$

always and equality outside the measure-zero degeneracy set of Appendix B.7. At the threshold  $\rho^*$  of Online Appendix Proposition S2.2, where  $\rho$  equals the larger eigenvalue of  $D\Pi$ , the intercept recursion is resonant and  $C(k)$  has a factor of  $k$ . The wealth autocovariance there is confluent,  $C(k)/V = (A + Bk)\rho^{*k} + E v_2^k$ , with  $v_2 = \theta_L \theta_H$  the smaller eigenvalue (its product with  $\rho^*$  is  $\det(D\Pi) = \theta_L \theta_H \rho^*$ ) and  $A + E = 1$ , because the form also holds at  $k = 0$ , where  $\beta_0 = 1$ . In the illustrative economy of Online Appendix S2,  $\rho^* = 5/7$ ,  $v_2 = 0.40$ , and  $(A, B, E) \approx (0.96, 0.10, 0.04)$ . Since  $k^{1/k} \rightarrow 1$ , the factor of  $k$  leaves the root rate unchanged.

A direct  $2 \times 2$  computation gives  $\det(D\Pi) = \theta_L \theta_H \rho$  and  $\text{tr}(D\Pi) = \rho(\theta_L + \theta_H) + (1 - \rho)\bar{\theta}$ . So  $q(D\Pi)$  is the larger root of  $v^2 - \text{tr}(D\Pi)v + \theta_L \theta_H \rho = 0$ , equal to  $\bar{\theta}$  at  $\rho = 0$  and  $\theta_H$  at  $\rho = 1$ , and rising strictly in between. The two eigenvalues are real and different on  $[0, 1]$ . For a  $2 \times 2$  matrix the discriminant  $\text{tr}(D\Pi)^2 - 4\theta_L \theta_H \rho$  equals the squared gap of the diagonal entries plus four times the product of the off-diagonal entries, and the off-diagonal entries of  $D\Pi$  are positive for  $\rho < 1$ . At  $\rho = 1$  the discriminant is  $(\theta_H - \theta_L)^2 > 0$ .

Implicit differentiation gives  $(2q(D\Pi) - \text{tr}(D\Pi)) \frac{d}{d\rho} q(D\Pi) = \check{\theta} q(D\Pi) - \theta_L \theta_H$ . Here  $\check{\theta} = p\theta_L + (1 - p)\theta_H$  is the reverse-weighted mean type, the derivative of the trace in  $\rho$ . The prefactor  $2q(D\Pi) - \text{tr}(D\Pi)$  is positive at the larger root. The right side is  $p(1 - p)(\theta_H - \theta_L)^2 > 0$  at  $\rho = 0$ , where  $q(D\Pi) = \bar{\theta}$ . Wherever the right side is positive,  $q(D\Pi)$  increases. A larger  $q(D\Pi)$  raises the right side further, so the right side stays positive and  $q(D\Pi)$  rises strictly on  $[0, 1]$ . The decay rate  $\max\{\rho, q(D\Pi)\}$  thus rises from  $\bar{\theta}$  toward one as  $\rho \rightarrow 1$ . The intercept channel  $\text{Cov}(w_t, \alpha_k(\theta_t))$  contains the mode  $\rho^k$  and components at the eigenvalues of  $D\Pi$ , which it inherits from the forcing  $b_{k-1}$ . The  $\rho$ -mode is the between-type channel, and it vanishes when types have no persistence. This proves part (i) of Online Appendix Proposition S2.1.

**Permanent types.** At  $\rho = 1$ ,  $\Pi = I$  gives  $b_k(s) = \theta_s^k$  and  $\alpha_k(s) = \mu_s(1 - \theta_s^k)$ . So  $h_k(s, w) = \mu_s(1 - \theta_s^k) + \theta_s^k w$ , and equation (16) reduces to  $C(k) = \sum_s \pi_s \theta_s^k v_s + V_B$ , matching Lemma B.1.

## B.6 Continuity and proofs

Online Appendix Proposition S2.1(ii) uses the following lemma.

**Lemma B.6** (Continuity of the Markovianization moment at  $\rho = 1$  and  $\rho = 0$ ). *Fix a horizon  $k$  and let the innovation density be bounded and continuous off a Lebesgue-null set, as with the exponential law. Write  $C_\rho^S(k)$  for the autocovariance of the Markovianization of the type-dependent economy with type persistence  $\rho$ . Then  $C_\rho^S(k) \rightarrow C_1^S(k)$  as  $\rho \rightarrow 1$ , and  $C_\rho^S(k) \rightarrow C_0^S(k)$  as  $\rho \rightarrow 0$ .*

*Proof.* Write  $Q_\rho$  for the joint stationary law of  $(w, \theta)$  at type persistence  $\rho$  and  $f_{s,\rho}$  for the conditional wealth density given type  $s$ , which exists because the innovation has a density. Tight-

ness: iterating equation (6) backward dominates every stationary wealth law, at every  $\rho$ , by the law of  $Z = \sum_{j \geq 1} \theta_H^{j-1} \varepsilon_{-j}$ , and  $\mathbb{E}[Z^2] < \infty$ , so  $\{Q_\rho\}$  is tight with uniformly integrable second moments. Limit identification: since the joint kernel is Feller in  $w$  and continuous in  $\rho$ , every weak limit point as  $\rho \rightarrow 1$  is invariant for the permanent-type chain and has type marginal  $\pi$ . Among the invariant laws of the permanent-type chain, only the  $\pi$ -mixture of the fixed-type stationary laws has type marginal  $\pi$ , hence  $Q_\rho \Rightarrow Q_1$ . Densities: the invariance identity gives  $\pi_{s'} f_{s', \rho}(x) = \sum_s \Pi_\rho[s, s'] \int f_\varepsilon(x - \theta_s w) Q_\rho(s, dw)$ . The integrand is bounded and continuous off a Lebesgue-null set and the limit slice laws are absolutely continuous, so weak convergence passes each integral to the limit, and  $f_{s, \rho}(x) \rightarrow f_{s, 1}(x)$  off a null set. The posterior  $\omega_\rho = p f_{H, \rho} / (p f_{H, \rho} + (1 - p) f_{L, \rho})$  then converges almost everywhere, because the denominator is positive on  $(0, \infty)$ .

Conditional means: define  $h_{j, \rho}$  by  $h_{0, \rho}(w) = w$  and

$$h_{j, \rho}(w) = \omega_\rho(w) \mathbb{E}[h_{j-1, \rho}(\theta_H w + \varepsilon)] + (1 - \omega_\rho(w)) \mathbb{E}[h_{j-1, \rho}(\theta_L w + \varepsilon)].$$

By induction  $0 \leq h_{j, \rho}(w) \leq \theta_H^j w + \sum_{i=1}^j \theta_H^{i-1} \mathbb{E} \varepsilon \leq \theta_H^j w + \mathbb{E} \varepsilon / (1 - \theta_H)$  uniformly in  $\rho$ , and  $h_{j, \rho} \rightarrow h_{j, 1}$  off a Lebesgue-null set by dominated convergence at each step. The moment:  $C_\rho^S(k) = \int w h_{k, \rho}(w) dF_\rho(w) - (\int w dF_\rho(w))^2$ , with  $F_\rho$  the wealth marginal and  $f_\rho = p f_{H, \rho} + (1 - p) f_{L, \rho}$  its density, which converges to  $f_1$  almost everywhere by the density step. The mean passes to the limit by weak convergence, because the domination by  $Z$  makes wealth uniformly integrable. For the cross moment,  $w h_{k, \rho}(w) f_\rho(w) \rightarrow w h_{k, 1}(w) f_1(w)$  off a Lebesgue-null set, and  $0 \leq w h_{k, \rho}(w) f_\rho(w) \leq (\theta_H^k w^2 + c w) f_\rho(w)$  with  $c = \mathbb{E} \varepsilon / (1 - \theta_H)$ . The dominating functions converge almost everywhere, and their integrals  $\theta_H^k \int w^2 dF_\rho + c \int w dF_\rho$  converge to the  $\rho = 1$  values by weak convergence and the uniform integrability of the second moments. Pratt's lemma (dominated convergence with converging dominators) then passes the cross moment to the limit, so  $C_\rho^S(k) \rightarrow C_1^S(k)$ . The limit  $\rho \rightarrow 0$  follows by the same steps. The limit chain at  $\rho = 0$  is geometrically ergodic with a unique invariant law (Appendix A.5), so the limit identification is immediate. Every later step applies with  $Q_1$  replaced by  $Q_0$ .  $\square$

*Proof of Lemma B.1.* With permanent types the cross-section is the mixture over  $\theta$  of the fixed-coefficient processes of Appendix B.2. Condition on the time-invariant type:

$$\mathbb{E}[w_t w_{t+k}] = \sum_\theta \pi_\theta \mathbb{E}[w_t w_{t+k} \mid \theta] = \sum_\theta \pi_\theta (\theta^k v_\theta + \mu_\theta^2),$$

using  $\text{Cov}(w_t, w_{t+k} \mid \theta) = \theta^k v_\theta$  and  $\mathbb{E}[w_t \mid \theta] = \mu_\theta$ . Subtract  $\mu^2$  and use  $\sum_\theta \pi_\theta \mu_\theta^2 - \mu^2 = V_B$  to get equation (12). Each within-type term  $\theta^k v_\theta$  is positive and falls in  $k$ , since  $\theta < 1$ . So  $\beta_k^T$  decreases to  $V_B/V$ .  $\square$

*Proof of Proposition 3.3, part (ii).* Any scale-dependent economy is a Markov chain in  $w$ . Under Assumptions 1 and 2, the argument of Proposition B.2 makes it irreducible, aperiodic and geometrically ergodic, so its autocovariance vanishes and  $\beta_k^S \rightarrow 0$ . By Lemma B.1,  $\beta_k^T \rightarrow V_B/V$ , which is positive because at least two types with different  $\theta$ , and so different  $\mu_\theta$ , have positive probability.  $\square$

*Proof of Corollary B.5.* Write  $V_B = p(1-p)(\mu_H - \mu_L)^2$  with  $\mu_\theta = (1/\lambda)/(1-\theta)$ , so  $V_B > 0$  exactly when  $\theta_H \neq \theta_L$  and  $p \in (0,1)$ . As  $\theta_H - \theta_L \rightarrow 0$ ,  $V_B \rightarrow 0$  while  $V_W \rightarrow v_{\theta_L} > 0$ , so the ratio vanishes. As  $p \rightarrow 0$  or  $p \rightarrow 1$ ,  $V_B \rightarrow 0$  while  $V$  tends to a single  $v_\theta > 0$ , so the ratio vanishes again. As  $\theta_H \rightarrow 1$ ,  $V_B$  grows like  $(1-\theta_H)^{-2}$  while  $V_W$  grows like  $(1-\theta_H)^{-1}$ , so  $V_B/V \rightarrow 1$ . Equation (14) substitutes the exponential moments into equations (10) and (11). The rate  $\lambda$  cancels in the ratio.  $\square$

## B.7 Local identification within the linear class

**Proposition B.7** (Local identification of  $\rho$  within the linear class). *In the two-type economy under Assumptions 1 and 2, with  $\rho > 0$  and the three rates  $v_1, v_2, \rho$  all different, the wealth autocovariance is, for every  $k \geq 1$ ,*

$$C(k) = c_1 v_1^k + c_2 v_2^k + c_3 \rho^k, \quad (17)$$

*with  $v_1, v_2$  the eigenvalues of  $D\Pi$ . When the three loadings are nonzero, the rates and loadings are recovered from the six autocovariances  $C(1), \dots, C(6)$ . The primitives then follow in two regimes. At  $\rho = 1$  the rate set is  $\{\theta_L, \theta_H, 1\}$  with the unit rate bearing the loading  $V_B$ , and identification is global, because no economy with  $\rho < 1$  has a unit rate. With  $\rho = 1$  maintained, the unit rate is one over-identifying restriction. For  $\rho < 1$  the full vector  $(\theta_L, \theta_H, p, \rho, \mathbb{E}\varepsilon, \sigma_\varepsilon^2)$  is locally identified outside a closed set of Lebesgue measure zero. Either way the wealth mobility profile identifies  $\rho$ , while  $\beta_0$  and  $\beta_1$  alone do not.*

**Recovering the rates and loadings.** By equation (17), for  $k \geq 1$  the wealth autocovariance is a sum of three geometric sequences  $C(k) = \sum_{i=1}^3 c_i r_i^k$ , with rates  $(r_1, r_2, r_3) = (v_1, v_2, \rho)$  and loadings  $(c_1, c_2, c_3)$ . Suppose no two rates coincide and no loading is zero. These three sequences are then linearly independent and satisfy a unique order-three linear recurrence  $C(k+3) = a_2 C(k+2) + a_1 C(k+1) + a_0 C(k)$ . The Hankel matrix  $[C(i+j-1)]_{i,j=1}^3$  built from

$C(1), \dots, C(5)$  is then nonsingular, its determinant being the product of the three loadings, the three rates, and  $\prod_{i < j} (r_i - r_j)^2$ . The rates are nonzero for  $\rho > 0$  because  $v_1 v_2 = \theta_L \theta_H \rho$ . So  $(a_0, a_1, a_2)$  are recovered from  $C(1), \dots, C(6)$ . The rates  $r_i$  are the roots of  $x^3 - a_2 x^2 - a_1 x - a_0$ , and the loadings solve a nonsingular Vandermonde system (Prony's method).

The Vandermonde system is singular exactly at the threshold  $\rho^*$  of Online Appendix Proposition S2.2, where  $\rho$  equals the larger eigenvalue of  $D\Pi$  and the wealth autocovariance has a  $k$ -weighted mode (Appendix B.5). The system is ill-conditioned near  $\rho^*$ , roughly where  $|\rho - \varrho(D\Pi)| < 0.01$ . The condition number of the Vandermonde matrix with rows  $r_i^0, r_i^1, r_i^2$  is about 90 at  $\rho = 0.5$  and about  $1.3 \times 10^3$  at  $\rho = 0.70$ . The Hankel solve before it is worse conditioned at both values. Away from it, the rate set  $\{v_1, v_2, \rho\}$  and the loadings are smooth functionals of  $C(1), \dots, C(6)$ , with nonsingular Jacobian wherever no two rates coincide and no loading is zero. So the rate set is recovered globally, as the roots of the order-three recurrence. The step from rates and loadings to the structural primitives is local (regime (b) below): away from  $\rho = 1$ , the rate set alone does not say which root is  $\rho$ .

**Regime (a): permanent types.** Let  $\rho = 1$ . By Lemma B.1, for  $k \geq 1$ :

$$C(k) = (1 - p) v_{\theta_L} \theta_L^k + p v_{\theta_H} \theta_H^k + V_B, \quad (18)$$

a three-mode wealth autocovariance with rates  $\{\theta_L, \theta_H, 1\}$  and loadings  $\{(1 - p)v_{\theta_L}, p v_{\theta_H}, V_B\}$ , all positive when  $\theta_L < \theta_H$  and  $p \in (0, 1)$ . The unit rate is the permanent component, and the two rates below one are  $\theta_L < \theta_H$ , which identifies the types. With  $v_\theta = \sigma_\varepsilon^2 / (1 - \theta^2)$ , the ratio of the two decaying loadings is:

$$\frac{p v_{\theta_H}}{(1 - p) v_{\theta_L}} = \frac{p}{1 - p} \cdot \frac{1 - \theta_L^2}{1 - \theta_H^2},$$

in which only  $p$  is unknown, so  $p \in (0, 1)$  is identified. The loading  $(1 - p)\sigma_\varepsilon^2 / (1 - \theta_L^2)$  then gives  $\sigma_\varepsilon^2$ , and the permanent loading  $V_B = p(1 - p)(\mu_{\theta_H} - \mu_{\theta_L})^2$  with  $\mu_\theta = \mathbb{E}\varepsilon / (1 - \theta)$  gives  $\mathbb{E}\varepsilon > 0$ . So  $(\theta_L, \theta_H, p, \mathbb{E}\varepsilon, \sigma_\varepsilon^2)$  are identified. With  $\rho = 1$  maintained, six recovered objects pin down five primitives, so the third rate equal to one is an over-identifying restriction, and the stationary mean  $\mu = p \mu_{\theta_H} + (1 - p) \mu_{\theta_L}$  gives a further one.

**Regime (b): mean-reverting types.** Let  $\rho < 1$ . Write  $\psi = (\theta_L, \theta_H, p, \rho, \mathbb{E}\varepsilon, \sigma_\varepsilon^2)$ . The stationary first and second moments of  $(w, \theta)$  solve linear fixed-point systems whose matrices are polynomial in  $\psi$ . Those matrices are invertible on the open region, because the spectral radii of  $\Pi^\top D$  and  $\Pi^\top D^2$  are below one. By Cramer's rule those moments are rational in  $\psi$ . So with the

polynomial coefficients  $\alpha_k, b_k$  of equation (15) the autocovariances built by equation (16) are rational in  $\psi$ , with denominators that do not vanish on:

$$\Psi = \{ 0 < \theta_L < \theta_H < 1, 0 < p < 1, 0 < \rho < 1, \mathbb{E}\varepsilon > 0, \sigma_\varepsilon^2 > 0 \}.$$

So the moment map  $M(\psi) = (C(1), \dots, C(6))$  is real-analytic on  $\Psi$ . I show that  $\partial M / \partial \psi$  has full column rank six on  $\Psi$  outside a measure-zero set, which gives local identification of  $\psi$  (Rothenberg, 1971). The determinant of  $\partial M / \partial \psi$  is real-analytic on  $\Psi$ , so by the identity theorem it either vanishes identically or only on a measure-zero set. One minor that is not identically zero suffices.

Near  $\rho = 1$  the three rates differ, and the map  $M \mapsto (v_1, v_2, c_1, c_2, c_3, \rho)$  is a local diffeomorphism, so  $\partial M / \partial \psi$  has the rank of the Jacobian of  $(v_1, v_2, c_1, c_2, c_3, \rho)$  in  $\psi$ . Evaluate the latter as  $\rho \rightarrow 1^-$ , with  $\rho$  ordered last. The rate  $\rho$  stays away from  $v_1 \rightarrow \theta_H$  and  $v_2 \rightarrow \theta_L$ , and the loadings approach the positive values in equation (18). The row  $\partial \rho / \partial \psi$  is the unit vector on the  $\rho$  coordinate, because  $\rho$  is itself a recovered rate. The roots of the recurrence are simple, so they are analytic in its coefficients, and the root equal to  $\rho$  at a point extends smoothly to a neighborhood of that point. What remains is the  $5 \times 5$  Jacobian of  $(\theta_L, \theta_H, p, \sigma_\varepsilon^2, \mathbb{E}\varepsilon) \mapsto (v_1, v_2, c_1, c_2, c_3)$  at  $\rho = 1$ , which is block lower-triangular. The derivatives of the rates,  $\partial(v_1, v_2) / \partial(\theta_L, \theta_H)$ , form a permutation matrix (at  $\rho = 1$ ,  $D\Pi = D$ , so  $v_1, v_2 = \theta_H, \theta_L$ ). The derivatives of the loadings  $c_1 = p v_{\theta_H}$ ,  $c_2 = (1 - p)v_{\theta_L}$  and  $c_3 = V_B$  in  $(p, \sigma_\varepsilon^2, \mathbb{E}\varepsilon)$  are:

$$\begin{pmatrix} v_{\theta_H} & \frac{p}{1 - \theta_H^2} & 0 \\ -v_{\theta_L} & \frac{1 - p}{1 - \theta_L^2} & 0 \\ (1 - 2p)(\mathbb{E}\varepsilon)^2 D_\mu^2 & 0 & 2p(1 - p) \mathbb{E}\varepsilon D_\mu^2 \end{pmatrix}, \quad D_\mu = \frac{1}{1 - \theta_H} - \frac{1}{1 - \theta_L},$$

whose determinant, by cofactor expansion along the last column, is:

$$2p(1 - p) \mathbb{E}\varepsilon D_\mu^2 \cdot \frac{\sigma_\varepsilon^2}{(1 - \theta_L^2)(1 - \theta_H^2)} \neq 0,$$

nonzero because  $\mathbb{E}\varepsilon, \sigma_\varepsilon^2 > 0$ ,  $p \in (0, 1)$ , and  $\theta_L \neq \theta_H$ . So this Jacobian is nonsingular at  $\rho = 1$ . The moment map is rational in  $\psi$  with denominators that stay nonzero at  $\rho = 1$ , where the spectral radius of  $\Pi^\top D$  is  $\theta_H < 1$ , and the three rates  $\theta_L, \theta_H$  and 1 differ there. By continuity the Jacobian is nonsingular for  $\rho$  in a left-neighborhood of one, and the corresponding  $6 \times 6$  minor of  $\partial M / \partial \psi$  is not identically zero on  $\Psi$ , which is convex, so connected. Hence  $\psi$  is locally identified at every  $\psi \in \Psi$  outside a closed measure-zero set, which contains the configurations

where two rates coincide or a loading vanishes. Proposition B.7 follows for the autocovariances.

The wealth mobility profile  $\beta_k = C(k)/C(0)$  identifies  $\rho$  as well. Scaling income by  $\tau > 0$  multiplies  $\mathbb{E}\varepsilon$  by  $\tau$ ,  $\sigma_\varepsilon^2$  and every  $C(k)$  by  $\tau^2$ , and leaves  $\rho$  and the wealth mobility profile unchanged. Two nearby parameter vectors with the same wealth mobility profile can therefore be rescaled to the same autocovariances, and the local identification above gives them the same  $\rho$ . The wealth mobility profile alone does not identify the scale of income. The stationary mean  $\mu$  or the variance  $V$  does.

## C The PSID sample

Wealth is household net worth in the PSID (Panel Study of Income Dynamics, 2023), defined as in Van Langenhove (2026a): financial assets, the main home and other real estate net of their mortgages, business equity, and pension accounts (individual retirement accounts and defined-contribution plans), less other debt. I exclude vehicles and impute bracketed components within their bracket.

Wealth ranks are percentiles 1 to 100 of net worth within a ten-year birth cohort and a wave (as in Van Langenhove, 2026c), with ties sharing their average rank and non-positive net worth at the bottom. The cohort is a ten-year span of the reference person's birth year, read from age at first observation.

The unit is the household, identified by its head-and-partner pair. A split or a new partnership starts a new household, so every wealth rank pair follows a stable head-and-partner pair. Each wealth rank pair has the PSID longitudinal family weight of its base wave, which only the weighted check below uses.

The wealth ranks are unweighted, and the sample includes the PSID's oversample of low-income households. A wealth rank-rank coefficient on unweighted wealth ranks therefore describes this sample, not the U.S. population. Weighting the wealth ranks changes it little: the ten-year wealth rank-rank coefficient is 0.68 instead of 0.69, and the two-year one stays 0.83.

The working-life sample has reference persons aged 30 to 54 in the biennial waves 2001 to 2023. Every wealth rank pair starts with a reference person aged 44 or less, so the subpopulation does not change with the horizon, and ten-year pairs start in 2001 to 2013. Table 3 counts the pairs by horizon and base wave. Online Appendix Tables S1 and S2 give the households left after each sample restriction and the PSID's income and wealth rank-rank coefficients by horizon.

Attrition by baseline wealth rank does not explain why the wealth rank-rank coefficient falls with the horizon. The ten-year pairs spread evenly across the baseline wealth deciles. Reweighting them to the two-year pairs' baseline gives a ten-year wealth rank-rank coefficient of 0.69, as before. On the balanced panel of 1,360 households observed at every wave to ten years, it is 0.70. Neither check rules out selection on wealth changes within deciles.

**Table 3: Wealth rank pairs by horizon and base wave**

| $k$ | 2001 | 2003 | 2005  | 2007  | 2009  | 2011  | 2013  | 2015  | 2017  | 2019  | 2021  | pairs  | households |
|-----|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|------------|
| 1   | 954  | 957  | 1,008 | 1,119 | 1,192 | 1,239 | 1,237 | 1,282 | 1,296 | 1,278 | 1,304 | 12,866 | 4,376      |
| 2   | 882  | 867  | 914   | 1,022 | 1,022 | 1,060 | 1,128 | 1,129 | 1,138 | 1,170 | –     | 10,332 | 3,596      |
| 3   | 818  | 836  | 847   | 871   | 875   | 953   | 999   | 993   | 1,064 | –     | –     | 8,256  | 2,991      |
| 4   | 772  | 766  | 742   | 783   | 798   | 879   | 899   | 935   | –     | –     | –     | 6,574  | 2,500      |
| 5   | 728  | 687  | 660   | 715   | 723   | 778   | 844   | –     | –     | –     | –     | 5,135  | 2,062      |

**Notes:** Wealth rank pairs at a horizon of  $k$  waves ( $2k$  years), by base wave. A dash marks a horizon that reaches past 2023. The last two columns give the total pairs and the number of households. The 4,630 households of the wealth mobility profile are the union over horizons.

## D The fitted model

### D.1 Specification

The wealth gradient is  $g(w) = 1 + d [\Phi((w - m)/s) - \bar{q}]$ , with  $\Phi$  the standard normal distribution function, location  $m = 6.01$  and scale  $s = 2.69$ . The centering  $\bar{q}$  is the cross-sectional mean of  $\Phi((w - m)/s)$  each period, so  $g$  averages one. Its smallest value in any economy of Figure 1 is 0.88. The type takes three equally spaced values and follows the chain of equation (1), and the memoryless shock is lognormal with mean one. With persistent labor income, Theorem 3.1 holds on wealth and age (Remark A.5), but Proposition 3.3 does not apply. The shares of Section 4.3 rest on no result of Section 3.

The labor income process of Section 4.1 puts all five income rank-rank coefficients within the PSID's sampling error (Online Appendix Table S2). Households enter with the autoregressive income component at its stationary law, whose standard deviation is  $\sigma_u = 0.38$ . Each period's income is one annual draw.

A household exits with probability  $\delta = 0.08$  per period, one over the 12.5 periods from age 30 to 55, and an entrant with zero wealth replaces it. Exit keeps the wealth distribution stationary at any type values (Remark A.4). The results are insensitive to the exit rate. At  $\delta = 0.06$  and 0.10 the economy without a type has a ten-year wealth rank-rank coefficient of 0.68 and 0.69, against 0.68 at 0.08. The widest permanent type stays mobility-consistent at both, with a share of type dependence of 55 percent, against 56.

I take the Pareto tail index of the stationary distribution to solve  $(1 - \delta) \varrho(\zeta) = 1$ , with  $\varrho(z)$  the spectral radius of the matrix  $\Pi_{jl} \mathbb{E}[(\theta_l g \eta)^z]$  at the top of the wealth gradient, where  $g$  is nearly flat. Beare and Toda (2022) prove this characterization for exactly multiplicative growth with reset. Here labor income and the wealth gradient make growth multiplicative only at the top. The index is at least 1.13 in every economy of Figure 1 that fits the top 10 wealth share, so every top wealth share is finite. Above the 99.5th percentile I replace the simulated households with a Pareto continuation at this index (Online Appendix S4.3). The continuation gives a top 10 wealth share of 0.74 both without a type and at the widest permanent type, against raw shares of 0.74 and 0.72 on ten million households.

On new simulation draws of the economies named in the text, every one keeps its leading mechanism, and no share moves by more than 1.4 points. Four economies at the edge of the mobility-consistent region fall just outside it, and none enters it.

## D.2 The shock dispersion

Wealth growth is the growth factor: next period's wealth net of new saving, divided by current wealth. In the PSID new saving is active saving, and the log factor's weighted standard deviation is 0.94 on the 9,325 household-waves with a positive factor. In the model new saving is labor income, and each wave's wealth report is given the PSID's rank reliability of 0.93, measured on households' mean ranks (Online Appendix S3), on households of the same ages and wealth ranks. At  $\bar{\sigma} = 0.65$  the economy without a type is as dispersed as the PSID, and the mobility-consistent economies have a growth factor's dispersion of 0.94 to 1.10. At 0.75 all lie above the PSID's range in Table 4, and at 0.55 all lie below it.

The robustness values of  $\bar{\sigma}$  are set at estimates of the growth factor's dispersion net of reporting error. Consecutive growth factors share a wealth report, so classical error of variance  $v$  in log wealth raises the log factor's variance by about  $2v$  and lowers its covariance one wave apart by about  $v$ . In the PSID the variance is 0.89, and the covariances are  $-0.09$ ,  $0.01$  and  $0.00$  one, two and three waves apart. Reading the reliability of 0.93 as the signal share of log wealth gives  $v = 0.167$  and a growth factor's dispersion net of error of 0.75. Reading the one-wave covariance,  $-0.087$ , as  $-v$  gives 0.85. With each factor a ratio of two wealth reports, a positive semidefinite covariance matrix of three consecutive growth factors needs 0.54 or more ( $v \leq 0.30$ ).  $\bar{\sigma} = 0.65$  does not depend on these corrections, because the model is held to the observed growth factor's dispersion.

## D.3 Matching the PSID's wealth growth

An economy is growth-factor-consistent when its log growth factor's dispersion and its autocorrelations at four, six and eight years lie in the PSID's ranges of Table 4. No economy with a type is fitted to these moments. Among the mobility-consistent economies, the growth-factor-consistent ones are the economy without a type, every type of width 0.3 or less, and the types of width 0.4 at a persistence of 0.95 or 0.96. Among them type dependence accounts for at most 46 percent. A wider type makes wealth growth too dispersed, and a nearly permanent one makes it too autocorrelated at eight years (Table 4).

**Table 4: The growth factor's autocorrelation in the PSID and in the permanent-type economies**

|                          | type dependence,<br>percent | growth factor's<br>log dispersion | autocorrelation |           |             | growth-factor-<br>consistent |
|--------------------------|-----------------------------|-----------------------------------|-----------------|-----------|-------------|------------------------------|
|                          |                             |                                   | four years      | six years | eight years |                              |
| PSID                     |                             | 0.94                              | 0.03            | 0.13      | 0.04        |                              |
| no type                  | 0                           | 0.94                              | 0.06            | 0.06      | 0.05        | yes                          |
| $\rho = 1, \Delta = 0.1$ | 18                          | 0.94                              | 0.06            | 0.06      | 0.05        | yes                          |
| $\rho = 1, \Delta = 0.2$ | 32                          | 0.94                              | 0.06            | 0.07      | 0.06        | yes                          |
| $\rho = 1, \Delta = 0.3$ | 41                          | 0.95                              | 0.08            | 0.09      | 0.08        | yes                          |
| $\rho = 1, \Delta = 0.4$ | 47                          | 0.97                              | 0.10            | 0.11      | 0.10        | no                           |
| $\rho = 1, \Delta = 0.5$ | 51                          | 0.99                              | 0.13            | 0.14      | 0.13        | no                           |
| $\rho = 1, \Delta = 0.6$ | 54                          | 1.02                              | 0.16            | 0.17      | 0.16        | no                           |
| $\rho = 1, \Delta = 0.7$ | 55                          | 1.04                              | 0.18            | 0.19      | 0.19        | no                           |
| $\rho = 1, \Delta = 0.8$ | 56                          | 1.07                              | 0.20            | 0.21      | 0.21        | no                           |

**Notes:** PSID row: the sample of Appendix C, with 95 percent cluster-bootstrap intervals of  $[-0.05, 0.10]$ ,  $[0.03, 0.25]$  and  $[-0.02, 0.09]$  at four, six and eight years, and  $[0.91, 0.98]$  for the growth factor's dispersion. Model rows: the mobility-consistent economy without a type and each mobility-consistent permanent type, with wealth observed at the PSID's rank reliability of 0.93. Autocorrelations are in levels, from one simulation draw. The growth factor's dispersion is a mean over ten draws, and the share of type dependence over three draws. The mobility-consistent economies with  $\rho < 1$  have autocorrelations of 0.04 to 0.22 and a growth factor's dispersion of 0.94 to 1.10.