

Online Appendix

Type Dependence and Scale Dependence in Top Wealth Concentration

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S1 A nonlinear, endogenous-saving illustration

In a model with a nonlinear, endogenous saving policy, I check the equivalence and the failure of the Markov property numerically. Proposition 3.3 covers only the linear law of equation (6), so here the positive floor and the memoryless twin’s vanishing autocorrelation rest on numerical results.

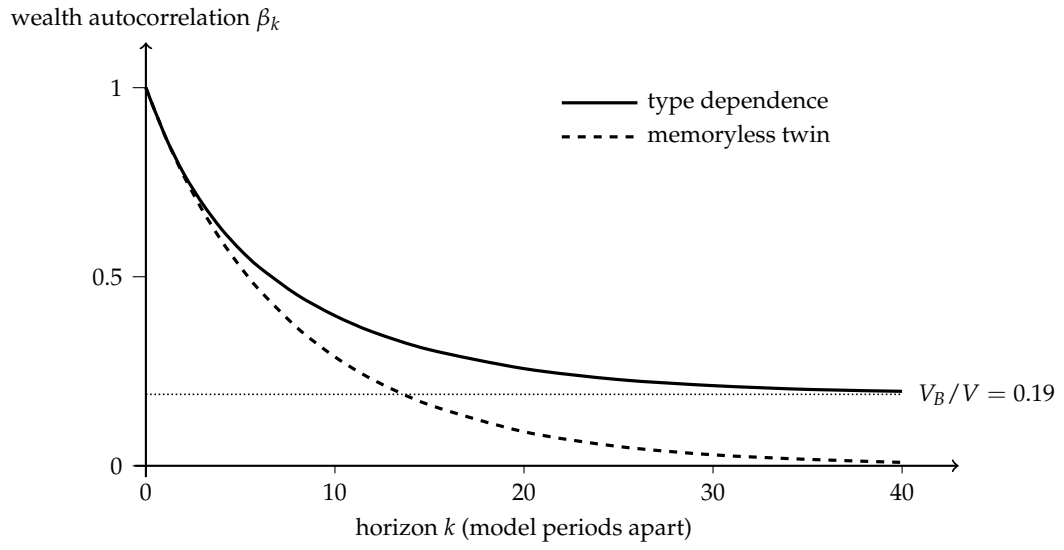
A household solves the stationary income-fluctuation problem:

$$\max \mathbb{E} \sum_{t \geq 0} \varsigma^t \frac{c_t^{1-\gamma}}{1-\gamma} \quad \text{subject to} \quad w_{t+1} = R_t w_t + y_t - c_t \geq 0,$$

with CRRA utility of curvature γ , discount factor ς and i.i.d. positive income y , lognormal and discretized at five Gauss–Hermite nodes. The gross return $R_t \in \{R_L, R_H\}$ is a persistent household type that follows the chain Π of equation (1), the type dependence of Fagereng et al. (2020). Wealth w_t is held at the start of period t , and the household chooses c_t after it sees R_t and y_t . The borrowing constraint and the curvature make the saving policy $w_{t+1} = \mathcal{S}(w_t, R_t, y_t)$ nonlinear in wealth, unlike the law of equation (6). I solve the policy by the endogenous grid method and compute the stationary joint law of (w, R) . The memoryless twin applies the same policy but draws the return each period from its wealth posterior $\varpi(w) = \Pr(R_H \mid w)$. It reproduces the policy’s transition law, but I do not show that it is an optimizing equilibrium under the redrawn return. The calibration is $\gamma = 2$, $\varsigma = 0.90$, $(R_L, R_H) = (1.00, 1.08)$, $p = \frac{1}{2}$, lognormal income with $\sigma_y = 0.35$, and permanent types ($\rho = 1$).

The two stationary laws agree on the grid to within 10^{-11} , and $\beta_1^T = \beta_1^S = 0.87$. The cross-section and every one-period statistic therefore coincide. The wealth mobility profiles then diverge, as in Proposition 3.3 (Figure S1). The type-dependent economy is non-ergodic, and its wealth autocorrelation levels off at the between-type share of wealth variance, $V_B/V = 0.19$. The memoryless twin is ergodic, and its autocorrelation decays to zero. With mean-reverting types ($\rho < 1$) both wealth mobility profiles decay to zero, as in the linear case, and at $\rho = 0.9$ the gap is a finite-horizon excess.

Figure S1: Wealth mobility profiles under a nonlinear saving policy, with permanent types



Notes: From the solved saving policy and its stationary distribution, at the calibration of Section S1. The two wealth mobility profiles coincide at $k = 1$. The dotted line is the between-type share of wealth variance, V_B/V .

S2 The wealth mobility profile: further results

The illustrative two-type economy of this section and of Appendix B.5 has $\theta_L = 0.5$, $\theta_H = 0.8$ and $p = 0.5$, and exponential income of rate $\lambda = 1$, which sets only the wealth scale. These values are illustrative, neither estimated nor calibrated to U.S. wealth data, and θ_H is chosen with θ_L so that $\rho^* = 5/7$.

S2.1 Decay rate under mean-reverting types

With mean-reverting types both wealth mobility profiles decay to zero (Appendix B.5 and Proposition B.2), so identification rests on their shape. At high persistence the wealth mobility profile decays more slowly than the memoryless twin's. At $\rho = 0.5$ the two differ by less than 0.03 at every horizon in this economy.

Proposition S2.1 (Decay rate and a finite-horizon gap). *In the two-type economy under Assumptions 1 and 2, with $p \in (0, 1)$, let $D = \text{diag}(\theta_L, \theta_H)$.*

- (i) *In the type-dependent economy with type persistence $\rho < 1$, the wealth autocovariance decays to zero at a geometric rate of at most $\max\{\rho, \varrho(D\Pi)\}$, with $\varrho(D\Pi)$ the within-type spectral radius. The rate rises in ρ , from $\bar{\theta}$ at $\rho = 0$ toward one as $\rho \rightarrow 1$ (Appendix B.5).*
- (ii) *At $\rho = 1$ there is a horizon K at which the type-dependent economy is strictly less mobile than its Markovianization, $\beta_K^T > \beta_K^S$, though $\beta_1^T = \beta_1^S$. The gap remains positive for ρ near one when the innovation density is bounded and continuous off a null set, as with the exponential law.*

As $\rho \rightarrow 0$ the type-dependent economy converges to its Markovianization, and the gap closes at every horizon.

S2.2 The dominance threshold

Moderate persistence barely changes the wealth mobility profile, because the type's persistence governs its tail only above a threshold. Away from ρ^* , the wealth autocovariance is a sum of three geometric modes (Proposition B.7). Two decay at the eigenvalues of $D\Pi$ and one at the persistence rate ρ .

Proposition S2.2 (Dominance threshold). *Set*

$$\rho^* = \frac{\bar{\theta} - \theta_L \theta_H}{1 - (\theta_L + \theta_H) + \bar{\theta}}. \quad (\text{S1})$$

Then ρ^* is the unique type persistence at which ρ equals the within-type spectral radius $\varrho(D\Pi)$, and $\rho^* \in (0, 1)$. For $\rho < \rho^*$ the persistence mode ρ^k is subdominant, and generically the long-run decay

rate of the wealth mobility profile is $\varrho(D\Pi)$. Within-type mean reversion sets this rate, which rises only from $\bar{\theta}$ to θ_H over the whole range of ρ . For $\rho > \rho^*$ the persistence mode dominates and the decay rate is generically ρ . In the illustrative economy, $\rho^* = 5/7$ and the radius rises from 0.65 to 0.80.

Proof of Proposition S2.1. Part (i) is proved in Appendix B.5.

For part (ii), at $\rho = 1$ Proposition 3.3 and Proposition B.2 give $\beta_k^T \rightarrow V_B/V > 0$ and $\beta_k^S \rightarrow 0$, so some horizon K has $\beta_K^T > \beta_K^S$. Near $\rho = 1$ the strict gap follows from continuity of the two fixed- K moments in ρ . The type-dependent economy's moment is built from $(D\Pi)^K$ and joint stationary moments that solve linear systems nonsingular up to and including $\rho = 1$, where the recursion reduces to Lemma B.1 (Appendix B.5). The Markovianization's moment is continuous at $\rho = 1$ under the stated density condition, by Lemma B.6. This continuity is of the fixed- K moment, not of the long-run limit, where ergodicity fails.

As $\rho \rightarrow 0$, $\Pi \rightarrow \mathbf{1}\pi^\top$ and the type draw becomes independent of the past. At $\rho = 0$ the type-dependent economy is therefore Markov, equal to its own Markovianization. Its fixed- k moment is continuous in ρ up to and including $\rho = 0$, because the stationary-moment systems of Appendix B.7 stay nonsingular there, and the Markovianization's is continuous at $\rho = 0$ by Lemma B.6. So the gap closes at every horizon. $\square$

Proof of Proposition S2.2. The value ρ is an eigenvalue of $D\Pi$ exactly when $\rho^2 - \text{tr}(D\Pi)\rho + \det(D\Pi) = 0$. Appendix B.5 gives $\det(D\Pi) = \theta_L\theta_H\rho$ and $\text{tr}(D\Pi) = \rho(\theta_L + \theta_H) + (1 - \rho)\bar{\theta}$. Dividing by $\rho > 0$ leaves a linear equation in ρ whose solution is equation (S1). The numerator is positive because $\bar{\theta} \geq \theta_L > \theta_L\theta_H$. The denominator $1 - (\theta_L + \theta_H) + \bar{\theta}$ is positive because it exceeds $(1 - \theta_L)(1 - \theta_H) > 0$. So $\rho^* > 0$. The inequality $\rho^* < 1$ reduces to $(1 - \theta_L)(1 - \theta_H) > 0$. The matched root is the larger of the two eigenvalues, since $\varrho(D\Pi)$ rises strictly in ρ from $\bar{\theta}$ at $\rho = 0$ to θ_H at $\rho = 1$, while the line ρ starts below it and ends above it (Appendix B.5). So ρ overtakes $\varrho(D\Pi)$ exactly once, at ρ^* , where the two eigenvalues are ρ^* and $\theta_L\theta_H$, equal to 0.71 and 0.40 in the illustrative economy. $\square$

S2.3 The number of types

The decay of the wealth mobility profile also reveals the number of types, because each type adds one geometric mode to the wealth autocovariance.

Proposition S2.3 (The number of types is identified). *Under Assumptions 1 and 2, let a type-dependent economy have J types, a type persistence $\rho > 0$, and the $J + 1$ rates $v_1, \dots, v_J, \rho$ all different. The wealth autocovariance $C(k) = \text{Cov}(w_t, w_{t+k})$ is then, for every $k \geq 1$, a sum of $J + 1$ geometric*

modes,

$$C(k) = \sum_{j=1}^J c_j v_j^k + c_{J+1} \rho^k, \quad (\text{S2})$$

with $v_1, \dots, v_J$ the eigenvalues of $D\Pi$, where $D = \text{diag}(\theta_1, \dots, \theta_J)$. The number of types equals the rank of the wealth-autocovariance Hankel matrix less one, $J = \text{rank } \mathcal{H} - 1$ with $\mathcal{H} = [C(i+j-1)]_{i,j \geq 1}$, whenever the loadings are nonzero.

Proof of Proposition S2.3. With J types and $\Pi = \rho I + (1-\rho)\mathbf{1}\pi^\top$, the recursion of Appendix B.5 holds with $D = \text{diag}(\theta_1, \dots, \theta_J)$ and the $J \times J$ matrix Π . The slope $b_k = (D\Pi)^k \mathbf{1}$ contributes the J eigenvalues $v_1, \dots, v_J$ of $D\Pi$. The intercept α_k evolves through Π , whose only non-unit eigenvalue is ρ . Its constant mode drops from the covariance as before, leaving the one further rate ρ . The matrix Π is diagonalizable, with $\Pi^m = \rho^m I + (1-\rho^m)\mathbf{1}\pi^\top$, so the repeated eigenvalue ρ adds the single term ρ^k and no term in $k\rho^k$. At $\rho = 1$ the constant mode is the between-type term of Lemma B.1, and it plays the role of the ρ mode. So, when the $J+1$ rates $v_1, \dots, v_J, \rho$ all differ, $C(k) = \sum_{j=1}^J c_j v_j^k + c_{J+1} \rho^k$ for $k \geq 1$, a wealth autocovariance of $J+1$ geometric modes. That sum is equation (S2).

By Kronecker's theorem the infinite Hankel matrix $\mathcal{H} = [C(i+j-1)]_{i,j \geq 1}$ has rank equal to the number of modes, so $J = \text{rank } \mathcal{H} - 1$ whenever the rates all differ and the loadings are nonzero. The order- $(J+1)$ recurrence, the rates, and the loadings then follow from $C(1), \dots, C(2J+2)$ exactly as in Appendix B.7. The closed-form inversion of the primitives is the two-type case. For general J the Jacobian at $\rho = 1$ is again block lower-triangular, with $\partial(v_1, \dots, v_J)/\partial(\theta_1, \dots, \theta_J)$ a permutation matrix. Local identification of the primitives then needs the loading block to be nonsingular. The two-type proof verifies it, and for $J > 2$ it is a condition, not a result. $\square$

S3 The PSID sample and wealth mobility profile

The rank reliability of 0.93 in Appendix D is a split-half correlation. A household's mean rank over its odd waves correlates 0.87 with its mean rank over its even waves, and 0.93 after the Spearman–Brown correction. The split-half also reflects true changes in rank, so it measures the stability of ranks, not reliability alone. That correlation is the reliability of a household's mean rank over its waves, while the model applies it to each wave's report. Under classical error the two-year wealth rank-rank coefficient, 0.83, bounds the reliability of one wave's rank from below.

Because the sample pools two decades of waves, the financial crisis included, stationarity is only an approximation. The longest lags rest on the fewest pairs (Table 3).

Table S1: Households left after each sample restriction of Appendix C, by wave

wave	in the household file	aged 30 to 54	positive weight	aged 44 or less	observed two years later
2001	4,224	2,322	2,211	1,329	954
2003	4,246	2,213	2,097	1,271	957
2005	4,369	2,212	2,212	1,322	1,008
2007	4,655	2,315	2,315	1,423	1,119
2009	4,912	2,416	2,416	1,514	1,192
2011	5,049	2,491	2,491	1,612	1,239
2013	4,993	2,452	2,452	1,661	1,237
2015	4,890	2,375	2,375	1,625	1,282
2017	4,901	2,391	2,391	1,676	1,296
2019	4,790	2,371	2,371	1,653	1,278
2021	4,679	2,364	2,364	1,638	1,304
2023	4,625	2,368	2,368	1,577	–

Notes: Each column keeps the households of the previous column that meet its restriction. Every household aged 30 to 54 has a within-cohort wealth rank. The first column counts households in the household-level file built from the PSID, not the families of its public release. The last column sums to the 12,866 two-year pairs of Table 3.

Table S2: Income and wealth rank-rank coefficients in the PSID, by horizon

years apart	income rank-rank		wealth rank-rank	
	coefficient	95 percent interval	coefficient	95 percent interval
2	0.86	[0.84, 0.87]	0.83	[0.82, 0.84]
4	0.83	[0.81, 0.84]	0.77	[0.75, 0.79]
6	0.79	[0.77, 0.81]	0.74	[0.72, 0.76]
8	0.77	[0.74, 0.80]	0.72	[0.69, 0.74]
10	0.76	[0.73, 0.79]	0.69	[0.66, 0.72]

Notes: Ranks within ten-year birth cohorts, on the sample of Appendix C, with income the household's labor income. The income coefficients use the pairs with an income rank at both ends, 12,855 at two years and 5,132 at ten. Intervals are from a household cluster bootstrap with 1,000 replicates, which resamples households once for all horizons, as Definition 5 requires.

S4 Heavy-tail machinery

S4.1 The within-type multiplier

Proposition B.4 adds within-type noise to the growth factor, $a_{it} = \theta_{it} \eta_{it}$, with $\eta_{it} > 0$ i.i.d. across households and time and independent of the types and the income innovations. The type then indexes the law the factor is drawn from, and its persistence keeps the meaning of Definition 1. The noise allows heavy tails, because without it a permanent type fixes the factor at a constant below one, which no Kesten condition covers. The factor ranges over $\mathbb{R}_+$, which lifts both the contraction and the baseline range $a_{it} \in (0, 1)$. With permanent types the recursion is i.i.d. within type, and the theory of Kesten (1973) and Goldie (1991) applies directly. With mean-reverting types the factor is Markov-modulated, and the tail follows from Beare and Toda (2022).

S4.2 Rank form under a Kesten law

Proof of Proposition B.4. Tails. Given the type, wealth follows the random affine recursion $w' = (\theta_s \eta)w + \varepsilon$. Under the stated conditions it has a unique stationary law whose upper tail is Pareto with index ζ_s , by the renewal theory of such recursions (Kesten, 1973; Goldie, 1991). The tail constant is positive because the innovation density keeps the recursion non-degenerate (Assumption 1). The function $\Lambda_s(q) = \log \mathbb{E}[(\theta_s \eta)^q]$ is convex with $\Lambda_s(0) = 0$ and $\Lambda'_s(0) = \mathbb{E}[\log(\theta_s \eta)] < 0$, and $\Lambda_H(q) - \Lambda_L(q) = q \log(\theta_H / \theta_L) > 0$ for $q > 0$. The root of Λ_H therefore comes first, $\zeta_H < \zeta_L$, and the high type has the heavier tail. The shape of Λ_s also gives $\mathbb{E}[(\theta_s \eta)^u] < 1$ for small $u > 0$, a condition that holds for any mixture of the two type laws.

Ergodicity of the scale-dependent class. Take the Lyapunov function $W(w) = 1 + w^u$ with this exponent u . Subadditivity of $x \mapsto x^u$ for $u \in (0, 1]$ gives $\mathbb{E}[W(w') \mid w] \leq 1 + \kappa w^u + \mathbb{E}[\varepsilon^u] \leq \kappa W(w) + L$ with $L = 1 - \kappa + \mathbb{E}[\varepsilon^u] < \infty$, a geometric drift. The chain also contracts with positive probability from every state: by Markov's inequality $\Pr(G(w, \eta') \leq \kappa^{1/(2u)}) \geq 1 - \kappa^{1/2} > 0$ uniformly in w , and the innovation falls below any fixed positive level with positive probability, independently of the factor. Finitely many such joint steps bring wealth below any level, one further draw of the positive-density innovation then reaches any open interval of $\mathbb{R}_+$, and compact sets are small. On $[0, M]$ the term $G(w, \eta') w$ is at most $\kappa^{1/(2u)} M$ with probability at least $1 - \kappa^{1/2}$, and the innovation density, bounded below on compact subsets of $(0, \infty)$, then keeps the kernel above a multiple of the uniform law on $[\kappa^{1/(2u)} M + 1, \kappa^{1/(2u)} M + 2]$, an interval inside $[0, M]$ once M is large. The chain is positive Harris recurrent and geometrically

ergodic, hence mixing, so the rank correlation of w_t and w_{t+k} tends to zero, its independence value. The Markovianization belongs to the class, because its factor law given w is the $\omega(w)$ -mixture of the two type laws and each satisfies the small- u moment condition.

Dominance. Couple the two type-conditional processes on a common stream $\{\eta_t, \varepsilon_t\}$. The stationary law is the law of the backward sum $\sum_{j \geq 1} (\prod_{i=1}^{j-1} \theta_s \eta_{-i}) \varepsilon_{-j}$, which converges almost surely under $\mathbb{E}[\log(\theta_s \eta)] < 0$. The first term is common to the two types, and every later term is strictly larger under θ_H because $\eta, \varepsilon > 0$. So $w^H > w^L$ almost surely under the coupling, and $\mathbb{E}[f(w^H)] > \mathbb{E}[f(w^L)]$ for every bounded strictly increasing f . The high-type stationary law therefore strictly dominates the low-type law in first order.

Rank limit. Under permanent types each household stays in one type-conditional chain, which mixes by the ergodicity argument applied at a small exponent. Given the type, w_t and w_{t+k} thus become independent as $k \rightarrow \infty$, and the joint law of (w_t, w_{t+k}) tends to the mixture $\sum_{\theta} \pi_{\theta} F_{\theta} \otimes F_{\theta}$. Write $m_{\theta} = \mathbb{E}[F(w) \mid \theta]$. The wealth rank-rank coefficient, the correlation of $F(w_t)$ and $F(w_{t+k})$, tends to $12 \text{Var}_{\pi}(m_{\theta})$, because $\mathbb{E}_{\pi}[m_{\theta}] = \mathbb{E}[F(w)] = 1/2$ and the variance of a uniform rank is $1/12$. The pooled distribution is strictly increasing on $\mathbb{R}_+$, since the innovation density is positive there. The coupling therefore gives $F(w^H) > F(w^L)$ almost surely and $m_H > m_L$. The limit is positive, and under scale dependence it is zero. The long-run wealth mobility profile in rank form therefore distinguishes type from scale dependence exactly as in the light-tailed baseline. $\square$

Proposition B.4 extends the rank form of the long-run floor (part (ii) of Proposition 3.3) to heavy tails. That wealth is non-Markov (part (i) of Proposition 3.3) needs its own argument there. Its proof uses the boundary $w_t = x/\theta_H$, above which the high type is infeasible, and the Kesten multiplier removes this boundary. Lemma S4.1 gives the argument.

Lemma S4.1 (Non-Markov wealth under the Kesten law). *Maintain the Kesten accumulation of Proposition B.4, with $a = \theta\eta$, a type $\theta \in \{\theta_L, \theta_H\}$ with type persistence ρ , and a memoryless multiplier $\eta > 0$ with a density. Let $h_s(x \mid w)$ be the density of next wealth $\theta_s \eta w + \varepsilon$ given type s and current wealth w . Suppose the log likelihood ratio $\log[h_H(x \mid w)/h_L(x \mid w)]$ is not the sum of a function of x and a function of w on a set of positive probability under the stationary law. Then for $\rho > 0$ wealth is non-Markov: on a set of positive probability $w_{t+2} \not\perp w_t \mid w_{t+1}$. At $\rho = 0$ wealth is Markov.*

Proof. The conditional law of w_{t+2} given (w_{t+1}, w_t) is the two-point mixture of $h_L(\cdot \mid w_{t+1})$ and $h_H(\cdot \mid w_{t+1})$, with weight on the high type the posterior $\Pr(\theta_{t+1} = \theta_H \mid w_{t+1}, w_t) = (1 - \rho)p + \rho \Pr(\theta_t = \theta_H \mid w_t, w_{t+1})$. Let $\omega(w) = \Pr(\theta_t = \theta_H \mid w_t = w)$. By Bayes' rule the

inner posterior at $w_{t+1} = x$ has log odds $\log[\varpi(w_t)/(1 - \varpi(w_t))] + \log[h_H(x | w_t)/h_L(x | w_t)]$. The first term depends on w_t alone. If the sum were constant in w_t for almost every x , the log likelihood ratio would be a function of x minus a function of w_t , which the hypothesis excludes. So for $\rho > 0$ the mixture weight varies with w_t on a set of positive probability. The two mixed laws differ because $\theta_L < \theta_H$ scales the multiplier η differently. So the conditional law of w_{t+2} depends on w_t given w_{t+1} on a set of positive probability, and wealth is non-Markov. At $\rho = 0$ the type θ_{t+1} is drawn from π independently of the past, so wealth is Markov. $\square$

The condition should hold for a lognormal multiplier with exponential income, although I do not prove it. There $h_s(x | w) = \int f_\eta(y) f_\varepsilon(x - \theta_s y w) dy$, the same base law with the multiplicative term scaled by $\theta_s w$. Without income the log ratio would be linear in $\log x - \log w$, the sum of a function of x and a function of w . The income integral breaks that form at moderate wealth, where it mixes x and w , and the log ratio is separable only in the limit far into the right tail. The support argument of Proposition 3.3 is the light-tailed limit of this likelihood-ratio condition, with the boundary at x/θ_H where $h_H(\cdot | w)$ vanishes and the ratio drops to zero.

S4.3 The Pareto continuation

In the economies of Section 4.3 a Pareto continuation with the model's implied tail index replaces the simulated cross-section above the 99.5th percentile (Appendix D). The top 10 and top 20 wealth shares combine the simulated body with the continuation.

The top shares of the simulated cross-section alone converge too slowly at these sample sizes. At the tail indices of these economies, 1.13 to 1.23, the sample mean converges at rate $N^{-0.12}$ to $N^{-0.19}$, and shares read from raw order statistics deep in the tail rest on a few hundred households. The fit surface they trace is also too rough for a derivative-free optimizer.

S5 The calibration

S5.1 Fitting the economies

The SCF's top 10 wealth share of household net worth, 0.74, is computed from the survey's five imputates with its weights (Board of Governors of the Federal Reserve System, 2023). In the economy without a type, the type's average and the slope minimize the sum of squared percent errors in the top 10 wealth share and the ten-year wealth rank-rank coefficient. The fit gives $\bar{\theta} = 0.58$ and a slope d of 0.84, which the secant step below refines to 0.85. Every other economy refits only the slope, by bisection on $[0, 4]$ with 24,000 households and then the secant method with 200,000.

In Figure 1, every solved economy's top 10 wealth share lies within 2 percent of the SCF's, and its Gini coefficient lies 4 to 7 percent below. Every solved economy there has a two-year wealth rank-rank coefficient of 0.77 to 0.83, and every mobility-consistent one 0.81 to 0.83, against the PSID's 0.83. Model moments average ten simulations of 200,000 households. The shape statistic of Definition 5 adds the covariance of the five coefficients across those ten simulations, divided by ten, to the PSID's cluster-bootstrap covariance.

A level test complements Definition 5. Its Wald statistic compares the five wealth rank-rank coefficients with the PSID's, against 11.07, the 95 percent point of a chi-square with five degrees of freedom. In Figure 1 it rejects every economy that is not mobility-consistent. Among the mobility-consistent ones it rejects only the types of width 0.4 with a persistence of 0.3 or less, the type of width 0.5 with a persistence of 0.96 and the types of width 0.7 and 0.8 with a persistence of 0.97. The smallest statistic is 7.07.

Table S3 compares three mobility-consistent economies with the data. Beside the share of type dependence it reports the type-related Shapley share, which also counts the type's dispersion as type dependence.

The Shapley decomposition switches off the five channels of Definition 4. Each Shapley value is relative to the fall from the economy to the one with all five channels off. The type-related Shapley share is the sum of the Shapley values of the type's persistence and dispersion. It differs from the share of type dependence by the interactions the Shapley values share out and by the type's dispersion at $\rho > 0$, which the share of type dependence counts as scale dependence.

Table S4 repeats the grid at the three other shock dispersions and under the three other labor income processes of Section 4.4, with $\bar{\theta}$ refitted each time. A lower $\bar{\theta}$ lowers the widest type

Table S3: Moments, tests and type dependence in three mobility-consistent economies

	U.S. data	No type	$\rho = 0.5$ $\Delta = 0.3$	$\rho = 1$ $\Delta = 0.8$
Top 10 wealth share	0.74	0.74	0.74	0.74
Top 20 wealth share	0.86	0.82	0.82	0.82
Gini coefficient	0.84	0.78	0.79	0.80
Wealth rank-rank coefficient, 2 years apart	0.83	0.82	0.82	0.83
Wealth rank-rank coefficient, 10 years apart	0.69	0.68	0.67	0.68
Wald statistic, five horizons	–	7.07	10.53	7.61
Wald statistic, shape of the wealth mobility profile	–	4.96	6.75	6.39
Growth factor's log dispersion	0.94	0.94	0.96	1.07
Slope of the wealth gradient, d	–	0.85	0.77	0.05
Share of type dependence, percent	–	0	13	56
Type-related Shapley share, percent	–	0	11	60

Notes: Economies of Definition 3. The last is the widest permanent type at which an economy exists. Each economy's slope targets the top 10 wealth share, and $\hat{\theta}$ and the slope of the economy without a type also target the ten-year wealth rank-rank coefficient (Section S5.1). Wealth inequality data are from the 2022 Survey of Consumer Finances and wealth mobility data from the PSID, with ranks within ten-year birth cohorts. Model moments are means over ten simulations of 200,000 households. The growth factor's log dispersion is measured as in Appendix D.2. The 95 percent points are 11.07 for the level test and 9.49 for the shape test of Definition 5.

width that keeps every type positive, to 0.7 at $\bar{\sigma} = 0.85$ and to 0.4 and 0.5 under the two labor income processes without a fixed effect.

Table S4: Type dependence under other shock dispersions and labor income processes

labor income process	$\bar{\sigma}$	mobility-consistent pairs where type dependence exceeds scale dependence	largest share of type dependence		
			mobility-consistent	growth-factor-consistent	ten-year coefficient
fitted to PSID income mobility	0.55	none	41	–	0.55–0.68
	0.65	$\rho \geq 0.99: \Delta \geq 0.3$ $0.95 \leq \rho \leq 0.98: \Delta \geq 0.4$	56	46	0.56–0.68
	0.75	$\rho \geq 0.95: \Delta \geq 0.2$ $0.8 \leq \rho \leq 0.9: \Delta \geq 0.3$	56	–	0.56–0.69
	0.85	$\rho = 1: \Delta \geq 0.1$ $0.71 \leq \rho \leq 0.95: \Delta \geq 0.2$ $\rho = 0.5: \Delta = 0.4$	56	–	0.61–0.69
Guvenen (2009)	0.69	$\rho = 1: \Delta \geq 0.5$ $\rho = 0.99: \Delta = 0.6$	49	41	0.58–0.68
	0.75	$\rho = 1: \Delta \geq 0.5$	50	–	0.59–0.68
Gaillard et al. (2024)	0.75	–	–	–	0.44–0.50
Gaillard and Wangner (2023)	0.75	–	–	–	0.48–0.54

Notes: Each row repeats the grid of Figure 1 for one labor income process and shock dispersion $\bar{\sigma}$, with $\bar{\theta}$ refitted (persistences: Section S5.3). The rows at $\bar{\sigma} = 0.65$ and 0.69 match the PSID growth factor’s dispersion. The third column gives, by the type’s persistence ρ , the type widths Δ . “None” means no mobility-consistent economy has more type dependence. Largest shares are in percent (Definition 4), with growth-factor-consistency as in Appendix D.3. A dash marks a row without such an economy. The last column is the range of the ten-year wealth rank-rank coefficient, against the PSID’s 0.69.

S5.2 The mobility-consistent region in detail

A wider type is mobility-consistent only at the ends of the persistence range. A type width of 0.4 is mobility-consistent at $\rho \leq 0.3$ and at $\rho \geq 0.95$, a type width of 0.5 at $\rho \geq 0.96$, a type width of 0.6 to 0.8 at $\rho \geq 0.97$, and a type width of 0.9 at $\rho = 0.98$. At the two pairs with a type width of 0.9 and a persistence of 0.99 or more, no economy matches the top 10 wealth share.

The pairs closest to a tie between type dependence and scale dependence are the types of width 0.3 with a persistence of 0.98 or more. Type dependence leads by 0.15 points at $\rho = 0.99$ and by 1.34 at the permanent type, and scale dependence leads by 1.35 at $\rho = 0.98$. On new simulation draws each keeps its sign (Appendix D.1).

S5.3 The robustness checks in detail

Guvenen (2009), in the first row of that paper’s Table 1, estimates on PSID male heads an annual persistence of 0.99 and standard deviations of 0.12 (innovation), 0.25 (transitory) and 0.24 (fixed effect). The labor income processes of Gaillard et al. (2024) and Gaillard and Wangner (2023) have a persistence of 0.95 and no fixed effect. Their innovation and transitory standard deviations are 0.16 and 0.20, and 0.20 and 0.15, reading the persistent standard deviation of

Gaillard and Wangner (2023) as that of its innovation. I use only their autoregressive and transitory components, mapped to two years as in Section 4.1. Under these two processes some economies have a tail index below one, but none is mobility-consistent, and wealth rank-rank coefficients need no finite moment.

The other variants' grids (Table S4) have nine persistences, without 0.96 to 0.99, except the Guvenen grid at its matched shock dispersion, which adds 0.98 and 0.99. In every variant with a mobility-consistent economy, type dependence reaches its largest share at a persistence of 0.98 or more.

At $\bar{\sigma} = 0.55$ the mobility-consistent types have a type width of 0.2 or less at every persistence, 0.3 at $\rho \leq 0.5$ and at $\rho = 1$, and 0.4 at $\rho \leq 0.1$. At 12 pairs (ρ, Δ) no economy matches the top 10 wealth share. At no shock dispersion, the baseline included, does the shape condition remove an economy.

Under the labor income process of Guvenen (2009) at $\bar{\sigma} = 0.69$, no economy matches the top 10 wealth share at 14 pairs, the widest types at the highest persistences.

S5.4 Calibrated models

Table 2 in Section 4.5 places the persistent types of calibrated models on the two axes of Figure 1. Table S5 gives the persistence and the spread each paper reports, and the paragraphs below explain how the models were selected and how each reported spread becomes a type width.

I searched the published and working-paper literature for calibrated models of the wealth distribution. A model enters the table if it is calibrated to moments of wealth inequality, has an exogenous persistent type in the return, the discount factor or the growth rate of wealth, and reports the type's persistence and spread. A model whose discount factors are set to match the marginal propensity to consume or the share of hand-to-mouth households does not enter.

Several kinds of model are left out. In some, households share one discount factor and one return (De Nardi, 2004; Hintermaier and Koeniger, 2011), or choose their projects or their returns (Quadrini, 2000; Koo, 2026). Some report the wealth distribution without targeting wealth inequality (Ma and Toda, 2021; Azzalini, Kondziella and Rácz, 2023; Daminato and Pistaferri, 2024; Makarski, Tyrowicz and Żoch, 2026; Li, Naknoi and Zhao, 2026), or add a discount-factor type only in an extension or an appendix (Hendricks, 2007b; Druedahl and Martinello, 2022; Straub, 2019; Piguillem, Ruffo and Trachter, 2026). Some set their discount factors to match the marginal propensity to consume or the share of hand-to-mouth households, alone or together

with the wealth distribution (Gong, 2023; Nakajima, 2023; Aguiar, Bils and Boar, 2025; Auclert, Rognlie and Straub, 2025; Propst, 2025; Briglia, 2026). The return states of Pugh (2021) and Sundram (2025), wide and mean-reverting within a few years, are discussed in the footnote to Section 4.5. The rest report no spread that converts: the entrepreneurial, investor and preference types listed in the footnote to Section 4.5, and Hagedorn, Manovskii and Mitman (2019), whose two discount factors target the wealth Gini but are not printed.

Some of the models in that footnote report a related statistic. Cagetti and De Nardi (2006) and Brüggemann and Mahone (2025) report the return on business wealth alone, and Gaillard et al. (2024) a return multiplier that scales both an interest rate the paper does not report and a wealth-dependent scale term. In Gomez (2025), the entrepreneurs' discount rate comes with a different elasticity of intertemporal substitution.

I convert each reported spread into the type width whose three-point type gives the same spread in the log two-year growth factor. The conversion assumes that households of every type consume the same share of their wealth, so a return gap of g a year is a gap of about $2g$ in the log two-year growth factor. A standard deviation is matched to the standard deviation of the log type, and a gap between two returns to the log gap between the highest and the lowest type.

A discount factor β , not the autocorrelation β_k of Appendix B.1, enters through the consumption share $1 - \beta$ of a household with log utility, so wealth grows in proportion to β and a gap in $\ln \beta$ per model period moves log wealth growth by the same amount. A quarterly gap counts eight times over two years. Where a paper prints the standard deviation of β , I divide it by the mean discount factor. Table S5 prints discount-factor gaps in levels. At the type's average of 0.58, a type width of 0.1 has a log standard deviation of 0.071. Each model's own consumption rule would move these type widths.

The annual persistence in Table S5 is reported in the source or derived from its reported transition matrix, retention probabilities or exit and entry rates. For a chain with more than two states it is the second-largest eigenvalue, and for Guvenen et al. (2023), whose lowest state is absorbing within a life, the leading eigenvalue of the other states. The two-year persistence is the square of the annual one, and 1 for a permanent type, which never changes within a life. The spread is the standard deviation of the return, the discount factor or the growth rate of wealth across households, or the gap between two of its values, in percentage points for a return or a growth rate. For a type with two values the spread is their gap. Each row uses the

paper's baseline calibration unless a note below says otherwise. The row of Benhabib, Bisin and Fernholz (2022) is their rank-based model with permanent growth types.

A return spread is before tax wherever the source states its tax treatment, except that of Cao and Luo (2017), which reports only after-tax returns. Their gap is in the return on net worth, 9.21 against 3.00 percent. Benhabib, Bisin and Luo (2019), Xavier (2021) and Albers, Kersting and Stieglitz (2026) do not state the tax treatment of their returns, which I read as before tax, and Kaymak, Leung and Poschke (2026) tax personal income, so their returns are before tax. A discount factor converts under a unit elasticity of intertemporal substitution, as in log utility. Hendricks (2007a), Brinca et al. (2016), Hubmer, Krusell and Smith (2021), Brinca et al. (2021), Ernst et al. (2024), Mahler and Yum (2024), De Nardi, Pashchenko and Porapakarm (2025) and Catherine, Lu and Paron (2026) have more curvature, which would narrow their type widths. The spreads of Ernst et al. (2024) and Gomez and Gouin-Bonenfant (2024) are in a discount rate, and the second is the gap between entrepreneurs with log utility and workers whose consumption rate equals their discount rate. Its growth state of firms has the same expected return as every other asset, so the table places it by these discount rates.

A range spans the interpretations of the source. For Hendricks (2007a) they span its three bequest motives. On the gap between its most and least patient types it would give a type width of up to 0.14. For Benhabib, Bisin and Zhu (2011) they are the lowest and highest standard deviations its calibrations produce. For Albers, Kersting and Stieglitz (2026) they are its calibration to 1800 and its simulated economy of 1914, which includes its scale dependence. "At most" marks an upper bound.

For Carroll et al. (2017) the two interpretations are its perpetual-youth and life-cycle models fitted to net worth. Both spreads are those of the seven equally weighted discount factors with which its perpetual-youth model approximates a uniform band. The spread of Carroll, Slacalek and Tokuoka (2014) places the same seven points within about 0.003 a quarter of the centre, the band the paper reports for net wealth across its euro-area countries without printing each country's. The fits of both papers to liquid assets are left out, since Figure 1 is fitted to net worth.

The spread of Guvenen et al. (2023) is the value-weighted standard deviation across households of their average annual return over ages 25 to 75, net of the average return at each age. This spread is a model outcome and can include the return's dependence on wealth. Its ability is fixed within a life, and the persistence is that of the productivity states that scale it.

The gap of Atkeson and Irie (2022) is in the expected growth of the level of wealth of a dynasty that holds a family firm against one that does not, net of its consumption. Table 2 uses this gap in the growth of the level. In expected log wealth growth the family-firm type grows 0.60 percentage points more slowly, a type width of 0.01. Their mechanism is the gap in volatility, which Figure 1 does not measure. The spread of Song (2025) is the unconditional standard deviation of its return component, $0.003/(1 - 0.96^2)^{1/2}$. Song (2025) also fixes for life whether a household holds the risky asset, a return gap of 1.88 percentage points and a type width of 0.02.

The spread of Kaymak, Leung and Poschke (2026) is the standard deviation of its return states at their population shares, which equals the 3.34 percent the paper reports for average returns. The growth rate of Benhabib, Bisin and Fernholz (2022) is a dynasty's, and its gap is in the growth of log wealth. The paper states no unit for it and its model period is a generation, so I read it per year, the larger of the two readings it allows. Krueger, Mitman and Perri (2016) draw a quarterly discount factor uniform on 0.9864 ± 0.0053 , which they discretize into five equally likely values without printing them, and the spread is that uniform's.

The gap of Catherine, Lu and Paron (2026) adds the 0.55-point gap in the return on every asset, whose tax treatment the paper states only as a combined fee and capital income tax of 2 percent, to the gap of 0.0260 in the discount factor, and their types also differ in risk aversion, and with it in their elasticity of intertemporal substitution. Ernst et al. (2024) print the patient shares of 1/4 and 1/5 without naming their base, and I read them within the skilled and the unskilled.

Mahler and Yum (2024) set their two discount factors at $\mu_\beta \pm \delta_\beta$ per two-year model period, with $\delta_\beta = 0.0284$, a gap of 0.0568 per two years and so 0.0284 a year. Their text calls 0.0284 the difference across types. The row of Brinca et al. (2021) is its German benchmark, and the row of De Nardi, Pashchenko and Porapakkarm (2025) its baseline. The discount factors of Erosa and Koreshkova (2007) are β_L , $2\beta_L$ and $3\beta_L$ per 30-year model period, drawn once a generation, and the row reads the standard deviation of their logarithm per year, weighted by the stationary shares of their transition matrix. The benchmark of Krusell et al. (2009) uses the discount factors and switching of Krusell and Smith (1998).

The two rows of Hubmer et al. (2026) are its models with a fixed return type and with a discount factor fixed within a generation. Each spread takes the type's dispersion the paper reports to be a standard deviation, as for its full model. The paper states no unit for its discount

factor's dispersion of 0.113, and the row takes it to be the standard deviation of the annual discount factor, divided by the basic model's 0.975, as the paper reports no mean.

Table S5: The inputs to Table 2: each paper’s persistence and spread

Model	Persistence, a year	Spread
<i>A. Impermanent types</i>		
Krusell and Smith (1998)	0.98	s.d. 0.0016 a quarter
Krusell et al. (2009)	0.98	s.d. 0.0016 a quarter
Cao and Luo (2017)	0.905	gap 6.21 pp
Bachmann et al. (2020)	0.98	s.d. 0.0009 a quarter
Hubmer, Krusell and Smith (2021)	0.992	s.d. 0.0050
Atkeson and Irie (2022)	0.932	gap 8.82 pp
Guvenen et al. (2023)	0.97	s.d. at most 4.10 pp
Song (2025)	0.96	s.d. 1.07 pp
Kaymak, Leung and Poschke (2026)	0.98	s.d. 3.34 pp
<i>B. Types permanent over a life</i>		
Erosa and Koreschkova (2007)	permanent	s.d. of $\ln \beta$ 0.0082 a year
Hendricks (2007a)	permanent	s.d. 0.0199 to 0.0403
Benhabib, Bisin and Zhu (2011)	permanent	s.d. 2.23 to 3.25 pp
Carroll, Slacalek and Tokuoka (2014)	permanent	s.d. 0.0017 a quarter
Suen (2014)	permanent	s.d. 0.0075
Brinca et al. (2016)	permanent	s.d. 0.0215
Krueger, Mitman and Perri (2016)	permanent	s.d. 0.0031 a quarter
Carroll et al. (2017)	permanent	s.d. 0.0038 to 0.0104 a quarter
Benhabib, Bisin and Luo (2019)	permanent	s.d. 2.69 pp
Brinca et al. (2021)	permanent	s.d. 0.0212
Xavier (2021)	permanent	s.d. 1.14 pp
Benhabib, Bisin and Fernholz (2022)	permanent	gap at most 2.14 pp
Ernst et al. (2024)	permanent	s.d. 0.0112
Gomez and Gouin-Bonenfant (2024)	permanent	gap 0.0050
Mahler and Yum (2024)	permanent	gap 0.0284 a year
De Nardi, Pashchenko and Porapakkarm (2025)	permanent	gap 0.1150
Albers, Kersting and Stieglitz (2026)	permanent	s.d. 1.04 to 1.90 pp
Catherine, Lu and Paron (2026)	permanent	gap 0.0315
Hubmer et al. (2026), return	permanent	s.d. 4.80 pp
Hubmer et al. (2026), discount factor	permanent	s.d. 0.1130

Notes: Each paper’s annual persistence and the spread of its persistent type, as the paper reports them or as computed from the values it reports. Table 2 converts each row to a two-year persistence and a type width at the type’s average of 0.58, under the conversion described in the text. The paragraphs below the table give the conventions and how each source is read.

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