

Supplemental Appendix

Saving Rate Heterogeneity under Wealth Rank Mobility

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S1 Further results

Households are ranked on wealth, $X_s = W_s$ at date s . I write $\mu_d = \beta_S^d$ for the start group's mean denominator, $\omega_{e|d} = \mathbb{P}(P_{t-1}^e \mid P_t^d)$ for the share of group d 's end members who started in group e , and $\Delta^d = R_{\text{syn}}^d - R_E^d$ for the synthetic bias.

S1.1 Bias of the saving rate

The saving rate is $\rho^d = \psi(R^d)$, so the bias in the gross ratio passes to the saving rate through ψ .

Proposition S1.1 (Bias of the saving rate). *For an affine $\psi(R) = \kappa_0 + \kappa_1 R$, $\rho_{\text{syn}}^d - \rho_E^d = \kappa_1 (R_{\text{syn}}^d - R_E^d)$. Whenever $\rho_E^d \neq 0$, the relative bias of the saving rate is the relative bias of the ratio times the elasticity of the saving rate with respect to the ratio,*

$$\frac{\rho_{\text{syn}}^d - \rho_E^d}{\rho_E^d} = \frac{\kappa_1 R_E^d}{\kappa_0 + \kappa_1 R_E^d} \left(\frac{\beta_E^d}{\beta_S^d} - 1 \right). \quad (\text{S1})$$

For a continuously differentiable ψ , $\rho_{\text{syn}}^d - \rho_E^d = \psi'(\xi^d) (R_{\text{syn}}^d - R_E^d)$ for some ξ^d between R_E^d and R_{syn}^d .

For the saving rate out of wealth, $\psi(R) = R - 1$, the elasticity is $R_E^d / (R_E^d - 1)$ and exceeds one for a small positive saving rate. The saving rate therefore has a larger relative bias than the gross ratio.

S1.2 The denominator gap by origin

Proposition S1.2 (Exact transition representation). *With $\bar{b}_{e \rightarrow d} = \mathbb{E}[B \mid P_t^d \cap P_{t-1}^e]$ the mean denominator of the households moving from e to d ,*

$$\beta_E^d - \beta_S^d = \sum_{e=1}^D \omega_{e|d} (\bar{b}_{e \rightarrow d} - \mu_d). \quad (\text{S2})$$

Replacing $\bar{b}_{e \rightarrow d}$ by the origin mean μ_e removes within-origin selection and gives Remark S1.1.

Remark S1.1 (Wealth rank mobility times dispersion). If $\bar{b}_{e \rightarrow d} = \mu_e$, then $\beta_E^d - \beta_S^d = \sum_e \omega_{e|d} (\mu_e - \mu_d)$. The off-diagonal transition mass measures wealth rank mobility, the spread of $\{\mu_e\}$ measures denominator dispersion, and the gap is their product. The gap is zero without wealth rank mobility and rises with both. Blanchet (2022) gives the continuous-time counterpart.

S1.3 The denominator gap by entry and exit

Partition the end group $P_t^d = I^d \sqcup U^d \sqcup F^d$ into the immobile households, the upward entrants and the downward entrants, with shares π_U^d and π_F^d . Partition the start group $P_{t-1}^d = I^d \sqcup \tilde{U}^d \sqcup \tilde{F}^d$ into the same immobile households and the upward and downward exiters, with shares π_U^d and π_F^d .

Proposition S1.3 (Flow decomposition of the gap). *With $\bar{b}_g$ the mean denominator of type g and I^d of positive mass,*

$$\beta_E^d - \beta_S^d = \pi_U^d(\bar{b}_{U^d} - \bar{b}_{I^d}) + \pi_F^d(\bar{b}_{F^d} - \bar{b}_{I^d}) - \pi_U^d(\bar{b}_{\tilde{U}^d} - \bar{b}_{I^d}) - \pi_F^d(\bar{b}_{\tilde{F}^d} - \bar{b}_{I^d}). \quad (\text{S3})$$

Under Assumption 1(c), $\bar{b}_{U^d} \leq \bar{b}_{I^d} \leq \bar{b}_{F^d}$. If the denominator is stochastically increasing in the destination rank within group d , then $\bar{b}_{\tilde{U}^d} \geq \bar{b}_{I^d} \geq \bar{b}_{\tilde{F}^d}$.

S1.4 A general denominator

Income, new resources or available resources can replace lagged wealth as the denominator B , with $\psi = \text{id}$. The start groups are then not level sets of B , and the rearrangement argument of Proposition 2.4 no longer applies.

Proposition S1.4 (Sign under a monotone denominator). *Suppose $0 < \mu_1 \leq \dots \leq \mu_D$ and that $\bar{b}_{e \rightarrow d} = \mu_e$ for every origin e including $e = d$. Then $\beta_E^1 \geq \beta_S^1$ and $\beta_E^D \leq \beta_S^D$. With $v_E^1, v_E^D > 0$ the synthetic method overstates the bottom, understates the top and compresses the saving-wealth gradient. The cumulative dominance (5) extends to this denominator. The identities of Propositions 2.3, S1.2 and S1.3 hold for any denominator.*

Across equal-sized groups the transition matrix is doubly stochastic, so $(\beta_E^d)_d$ is majorized by $(\mu_e)_e$ and both endpoint signs follow. The PSID does not support the no-selection condition $\bar{b}_{e \rightarrow d} = \mu_e$, and the exact magnitude of Proposition 2.3 holds for lagged wealth alone.

S1.5 Consistency and inference

Proposition S1.5 (Strong consistency). *Let $\{(X_{i,t-1}, X_{i,t}, N_i, B_i)\}_{i=1}^n$ be independent and identically distributed. Form $\hat{P}_s^d$ from the empirical quantiles of $\{X_{i,s}\}$ and let $\hat{R}_E^d, \hat{R}_{\text{syn}}^d, \hat{R}_S^d$ be the sample ratios of means. Under Assumption 1, $\hat{R}_E^d \rightarrow R_E^d$, $\hat{R}_{\text{syn}}^d \rightarrow R_{\text{syn}}^d$ and $\hat{R}_S^d \rightarrow R_S^d$ almost surely, hence $\hat{\Delta}^d \rightarrow \Delta^d$ almost surely.*

Remark S1.2 (In-sample identities and sampling uncertainty). Propositions 2.3, S1.2 and S1.3 hold at the realized group aggregates in any sample. Each $\hat{\Delta}^d$ is a smooth function of sample

means and sample quantiles. Suppose $\mathbb{E}[N^2], \mathbb{E}[B^2] < \infty$, the density of X_s is positive at the group boundaries and the conditional means of N and B given X_s are continuous there. The delta method then gives $\sqrt{n}(\hat{\Delta}^d - \Delta^d) \Rightarrow \mathcal{N}(0, V_d)$ (van der Vaart, [1998](#), Ch. 20–21), with the estimated boundaries entering V_d through that density. Household block bootstrap intervals in this supplement use 1,000 replicates and the 90 percent percentile.

S2 Proofs

Proof of Corollary 2.1

Write $\omega_{e|d} = \mathbb{P}(P_{t-1}^e \mid P_t^d)$ for the share of group d 's end members who started in group e . If $\omega_{d|d} = 1$ for every d , then $P_t^d \subseteq P_{t-1}^d$ up to probability zero, and the equal mass $1/D$ forces $P_t^d = P_{t-1}^d$ up to probability zero, so $\beta_E^d = \mathbb{E}[B \mid P_t^d] = \mathbb{E}[B \mid P_{t-1}^d] = \beta_S^d$ and $\Delta^d = 0$. Conversely, suppose the end-group and synthetic rates coincide at every d , so $\Delta^d = 0$. Then $\nu_E^d \neq 0$ forces $\beta_E^d = \beta_S^d$ by Proposition 2.3, so $g_d = 0$ and $G_k = 0$ for all k . Equation (5) then holds with equality at every k . By the strictness clause of Proposition 2.4 this forces $\bigcup_{d \leq k} P_t^d = \bigcup_{d \leq k} P_{t-1}^d$ up to probability zero for all k . Hence $P_t^d = P_{t-1}^d$ up to probability zero for every d , and wealth rank mobility is absent. $\square$

Proof of Proposition 2.2

Partition $P_t^d = I^d \sqcup U^d \sqcup F^d$. Averaging over the partition gives $\nu_E^d = \sum_g \pi_g^d \nu_g^d$ and $\beta_E^d = \sum_g \pi_g^d \beta_g^d$, so with $w_g^d = \pi_g^d \beta_g^d / \beta_E^d$,

$$R_E^d = \frac{\sum_g \pi_g^d \nu_g^d}{\beta_E^d} = \sum_g \frac{\pi_g^d \beta_g^d}{\beta_E^d} \cdot \frac{\nu_g^d}{\beta_g^d} = \sum_g w_g^d R_g^d,$$

and $\sum_g w_g^d = 1$ by construction. Subtracting one from each side and using that the weights sum to one gives $s_E^d = \sum_g w_g^d s_g^d$. Replacing each s_g^d by $s_I^d + (s_g^d - s_I^d)$ under the same weights gives (2). The weights lie in $[0, 1]$ wherever every type's mean lagged wealth is nonnegative, and the identity holds with a weight outside $[0, 1]$ where an entrant type has a nonpositive mean. $\square$

Proof of Proposition 2.3

The functionals R_{syn}^d and R_E^d share the numerator ν_E^d , so

$$\Delta^d = \frac{\nu_E^d}{\beta_S^d} - \frac{\nu_E^d}{\beta_E^d} = \nu_E^d \frac{\beta_E^d - \beta_S^d}{\beta_E^d \beta_S^d},$$

with positive denominators by hypothesis. Hence $\text{sign } \Delta^d = \text{sign}(\nu_E^d) \text{sign}(\beta_E^d - \beta_S^d)$. For $\nu_E^d \neq 0$, dividing by $R_E^d = \nu_E^d / \beta_E^d$ gives $\Delta^d / R_E^d = (\beta_E^d - \beta_S^d) / \beta_S^d = \beta_E^d / \beta_S^d - 1$, equivalently $R_{\text{syn}}^d / R_E^d = \beta_E^d / \beta_S^d$. $\square$

Lemma S2.1 (Extremal denominators). *Let B be integrable with atomless distribution and $p \in (0, 1)$. Among all $A \in \mathcal{F}$ with $\mathbb{P}(A) = p$, the conditional mean $\mathbb{E}[B \mid A]$ is maximized by the upper*

level set $\{B > Q_{1-p}^B\}$ and minimized by the lower level set $\{B \leq Q_p^B\}$. Each extremum is unique up to probability zero, with strict inequality unless A equals the extremal set up to probability zero.

The lemma is the bathtub principle of Lieb and Loss (2001) and the extremal form of the rearrangement inequality of Hardy, Littlewood and Pólya (1952).

Proof of Lemma S2.1

Let $c = Q_{1-p}^B$, so $A^+ = \{B > c\}$ with $\mathbb{P}(A^+) = p$ by atomlessness. For any A with $\mathbb{P}(A) = p$, the sets $A^+ \setminus A$ and $A \setminus A^+$ have the common probability m . On the first $B > c$, on the second $B \leq c$, so

$$\mathbb{E}[B \mathbf{1}_{A^+}] - \mathbb{E}[B \mathbf{1}_A] = \mathbb{E}[B \mathbf{1}_{A^+ \setminus A}] - \mathbb{E}[B \mathbf{1}_{A \setminus A^+}] \geq c m - c m = 0.$$

Dividing by p gives $\mathbb{E}[B \mid A^+] \geq \mathbb{E}[B \mid A]$. If $m > 0$, then $\mathbb{P}(B = c) = 0$ makes $B > c$ strict on $A^+ \setminus A$, so the inequality is strict. If $m = 0$, then $A = A^+$ up to probability zero. The minimizer claim follows by replacing B with $-B$. $\square$

Proof of Proposition 2.4

Each group has mass $1/D$, so $\sum_{d \leq k} \beta_S^d = D \mathbb{E}[B \mathbf{1}_{\cup_{d \leq k} P_{t-1}^d}]$ and $\sum_{d \leq k} \beta_E^d = D \mathbb{E}[B \mathbf{1}_{\cup_{d \leq k} P_t^d}]$. By Assumption 1(c), $\cup_{d \leq k} P_{t-1}^d = \{B \leq Q_{k/D}^B\}$ is the lower level set of mass k/D , and $\cup_{d \leq k} P_t^d$ has the same mass. The minimizer form of Lemma S2.1 gives $\mathbb{E}[B \mathbf{1}_{\cup_{d \leq k} P_{t-1}^d}] \leq \mathbb{E}[B \mathbf{1}_{\cup_{d \leq k} P_t^d}]$, hence (5), with $G_k = \sum_{d \leq k} (\beta_E^d - \beta_S^d) \geq 0$. That minimum is over every set of mass k/D . Comparing the lower level set with the union of the k end-groups of smallest denominator mean therefore gives the ascending-order inequality $\sum_{j \leq k} \mu_j \leq \sum_{j \leq k} \beta_E^{(j)}$. Here $\mu_1 \leq \dots \leq \mu_D$ are the start-group denominators in ascending order, their stated order under Assumption 1(c). The sequence $\beta_E^{(1)} \leq \dots \leq \beta_E^{(D)}$ reorders the end-group denominators. This ordering is the majorization $(\beta_E^d)_d \prec (\beta_S^d)_d$, which is the mean-preserving-spread claim. At $k = D$ both sides equal $D \mathbb{E}[B]$, so $G_D = 0$. The strictness clause of the lemma makes the level- k inequality strict iff $\cup_{d \leq k} P_t^d$ differs from the lower level set up to probability zero. $\square$

Proof of Corollary 2.5

The case $k = 1$ of (5) gives $\beta_S^1 \leq \beta_E^1$. Subtracting the case $k = D - 1$ from the equality at $k = D$ gives $\beta_S^D \geq \beta_E^D$. With $\nu_E^1, \nu_E^D > 0$ and positive denominators, Proposition 2.3 gives $R_{\text{syn}}^1 \geq R_E^1$ and $R_{\text{syn}}^D \leq R_E^D$. Subtracting, $R_{\text{syn}}^D - R_{\text{syn}}^1 \leq R_E^D - R_E^1$. Writing $G_E = R_E^D - R_E^1$ and $W = (R_E^D - R_{\text{syn}}^D) + (R_{\text{syn}}^1 - R_E^1) \geq 0$, the synthetic saving-wealth gradient equals $G_E - W$,

negative when $0 < G_E < W$. For nondecreasing ψ the saving rate $\rho = \psi(R)$ preserves the orderings.

The cumulative dominance moreover signs every monotone-weighted sum. With $g_d = \beta_E^d - \beta_S^d$, $G_k = \sum_{d \leq k} g_d$, and $G_0 = 0$, summation by parts gives

$$\sum_{d=1}^D c_d g_d = \sum_{d=1}^D c_d (G_d - G_{d-1}) = c_D G_D - \sum_{d=1}^{D-1} (c_{d+1} - c_d) G_d = \sum_{k=1}^{D-1} (c_k - c_{k+1}) G_k, \quad (\text{S4})$$

using $G_0 = 0$ and $G_D = 0$. Proposition 2.4 gives $G_k \geq 0$. The sum is then nonpositive for nondecreasing c and nonnegative for nonincreasing c . $\square$

Proof of Proposition S1.1

For affine $\psi(R) = \kappa_0 + \kappa_1 R$, $\rho_{\text{syn}}^d - \rho_E^d = \kappa_1 (R_{\text{syn}}^d - R_E^d)$ exactly. Dividing by $\rho_E^d = \kappa_0 + \kappa_1 R_E^d$ and substituting $R_{\text{syn}}^d - R_E^d = R_E^d (\beta_E^d / \beta_S^d - 1)$ from (3) gives (S1). For continuously differentiable ψ , the mean value theorem on the interval with endpoints R_E^d and R_{syn}^d gives $\psi(R_{\text{syn}}^d) - \psi(R_E^d) = \psi'(\xi^d)(R_{\text{syn}}^d - R_E^d)$ for some ξ^d strictly between them. $\square$

Proof of Proposition S1.2

The family $\{P_{t-1}^e\}_e$ partitions Ω up to probability zero. Conditioning on P_t^d and applying the law of total expectation,

$$\beta_E^d = \sum_{e=1}^D \mathbb{P}(P_{t-1}^e \mid P_t^d) \mathbb{E}[B \mid P_t^d \cap P_{t-1}^e] = \sum_{e=1}^D \omega_{e|d} \bar{b}_{e \rightarrow d},$$

omitting origins with $\omega_{e|d} = 0$. Subtracting $\beta_S^d = \mu_d = \sum_e \omega_{e|d} \mu_d$ gives (S2). $\square$

Proof of Proposition S1.3

Partition $P_t^d = I^d \sqcup U^d \sqcup F^d$ and $P_{t-1}^d = I^d \sqcup \tilde{U}^d \sqcup \tilde{F}^d$ with common immobile households $I^d = P_t^d \cap P_{t-1}^d$. Averaging B over each partition, subtracting $\bar{b}_{I^d}$ using that shares sum to one, and differencing the two lines removes $\bar{b}_{I^d}$ and gives (S3). Under Assumption 1(c), B is increasing in X_{t-1} , so the start thresholds in X_{t-1} map to thresholds in B . Upward entrants have X_{t-1} below the lower threshold of group d , hence $\bar{b}_{U^d} \leq \bar{b}_{I^d}$, and downward entrants above the upper threshold, hence $\bar{b}_{F^d} \geq \bar{b}_{I^d}$. The exiters and immobile households all lie in P_{t-1}^d , so thresholds do not separate them. If the denominator is stochastically increasing in the destination rank, the denominator distribution of upward exiters first-order dominates that of immobile households, which dominates that of downward exiters, giving $\bar{b}_{\tilde{U}^d} \geq \bar{b}_{I^d} \geq \bar{b}_{\tilde{F}^d}$. $\square$

Proof of Proposition S1.4

Under $\bar{b}_{e \rightarrow d} = \mu_e$, Proposition S1.2 gives $\beta_E^d = \sum_e \omega_{e|d} \mu_e$, a weighted average of $\{\mu_e\}$. Since $\mu_1 \leq \mu_e \leq \mu_D$, $\beta_E^1 \geq \mu_1 = \beta_S^1$ and $\beta_E^D \leq \mu_D = \beta_S^D$, so with $v_E^1, v_E^D > 0$ the saving rate signs and saving-wealth gradient comparison follow as in Corollary 2.5. The kernel is doubly stochastic across equal-sized groups, since $\sum_e \omega_{e|d} = 1$ and $\sum_d \omega_{e|d} = \sum_d \mathbb{P}(P_{t-1}^e | P_t^d) = D \sum_d \mathbb{P}(P_{t-1}^e \cap P_t^d) = D \mathbb{P}(P_{t-1}^e) = 1$. Hence $(\beta_E^d)_d$ is a doubly stochastic image of $(\mu_e)_e$ and is majorized by it. Majorization gives $\sum_{d \leq k} \beta_E^{(d)} \geq \sum_{d \leq k} \mu_d$ for the k smallest end-group means, and the index-order sum $\sum_{d \leq k} \beta_E^d$ is at least the sum of the k smallest, so $\sum_{d \leq k} \beta_E^d \geq \sum_{d \leq k} \mu_d = \sum_{d \leq k} \beta_S^d$. $\square$

Proof of Proposition S1.5

Fix d . Write $I = (Q_{(d-1)/D}^t, Q_{d/D}^t]$ for the true end interval in X_t , $\hat{I}$ for its empirical counterpart, and $\hat{n}_n = \frac{1}{n} \sum_i B_i \mathbf{1}\{X_{i,t} \in \hat{I}\}$, $\hat{m}_n = \frac{1}{n} \sum_i \mathbf{1}\{X_{i,t} \in \hat{I}\}$. Under Assumption 1(a) each population quantile is unique, so $\hat{Q}_\alpha^t \rightarrow Q_\alpha^t$ almost surely (Serfling, 1980, §2.3). Fix $\varepsilon > 0$ and let J_ε be the union of the ε -bands about the two boundaries of I . On the almost-sure event that both empirical quantiles lie within ε of their limits, $\hat{I} \triangle I \subseteq J_\varepsilon$, so

$$\left| \hat{n}_n - \frac{1}{n} \sum_i B_i \mathbf{1}\{X_{i,t} \in I\} \right| \leq \frac{1}{n} \sum_i B_i \mathbf{1}\{X_{i,t} \in J_\varepsilon\},$$

using $B > 0$. By the strong law the right side converges to $\mathbb{E}[B \mathbf{1}\{X_t \in J_\varepsilon\}]$ and the fixed-boundary average to $\mathbb{E}[B \mathbf{1}\{X_t \in I\}]$, so $\limsup_n |\hat{n}_n - \mathbb{E}[B \mathbf{1}\{X_t \in I\}]| \leq \mathbb{E}[B \mathbf{1}\{X_t \in J_\varepsilon\}]$ almost surely. Letting $\varepsilon \downarrow 0$, atomlessness makes the boundary have probability zero and dominated convergence sends the bound to zero, so $\hat{n}_n \rightarrow \mathbb{E}[B \mathbf{1}\{X_t \in I\}]$, and with $B \equiv 1$, $\hat{m}_n \rightarrow 1/D$. The continuous mapping theorem gives $\hat{\beta}_E^d \rightarrow \beta_E^d$. Replacing B by the integrable N gives $\hat{v}_E^d \rightarrow v_E^d$, and the start-group analogues converge by the same argument. A final application of the continuous mapping theorem to the ratios yields $\hat{R}_E^d \rightarrow R_E^d$, $\hat{R}_{\text{syn}}^d \rightarrow R_{\text{syn}}^d$, $\hat{R}_S^d \rightarrow R_S^d$, and $\hat{\Delta}^d \rightarrow \Delta^d$ almost surely. $\square$

Proof of Proposition S3.1

I first record a Gaussian tilting identity. For jointly normal (U, V) with zero means, common variance σ^2 , and covariance γ ,

$$\mathbb{E}\left[e^U \mathbf{1}\{V > c\}\right] = e^{\sigma^2/2} \Phi\left(\frac{\gamma - c}{\sigma}\right). \quad (\text{S5})$$

Tilting the measure by $e^U/\mathbb{E}[e^U]$ shifts the mean of V by $\text{Cov}(U, V) = \gamma$ and leaves the variances unchanged. The tail probability of V under the tilt is therefore $\Phi((\gamma - c)/\sigma)$, and $\mathbb{E}[e^U] = e^{\sigma^2/2}$ gives (S5).

Now $B = e^{x_{t-1}}$, and each group has mass $1/D$, so $\beta^d = D \mathbb{E}[B \mathbf{1}\{\cdot\}]$. The start group conditions on x_{t-1} , where the relevant covariance is $\text{Var}(x_{t-1}) = \sigma^2$. Its band in x_{t-1} runs from $\sigma z_{(d-1)/D}$ to $\sigma z_{d/D}$. Applying (S5) at the two edges and differencing,

$$\beta_S^d = D e^{\sigma^2/2} \left[\Phi(\sigma - z_{(d-1)/D}) - \Phi(\sigma - z_{d/D}) \right].$$

The end group conditions on x_t , where the relevant covariance is $\text{Cov}(x_{t-1}, x_t) = \theta\sigma^2$, so the same steps give $\beta_E^d = D e^{\sigma^2/2} [\Phi(\theta\sigma - z_{(d-1)/D}) - \Phi(\theta\sigma - z_{d/D})]$. The factor $D e^{\sigma^2/2}$ cancels in the ratio, giving (S7). The top group sets $z_{d/D} = +\infty$, so $\Phi(\theta\sigma - z_{d/D}) = 0$. $\square$

Proof of Proposition S3.2

The numerator $\Phi(\theta\sigma - z_{1-p})$ has θ -derivative $\sigma \phi(\theta\sigma - z_{1-p}) > 0$, and the denominator $\Phi(\sigma - z_{1-p})$ does not depend on θ . The top wedge is therefore strictly increasing in θ . At $\theta = 1$ the numerator and denominator coincide, giving one. For $\theta < 1$, $\theta\sigma - z_{1-p} < \sigma - z_{1-p}$, and Φ is strictly increasing, so the wedge is below one. $\square$

S3 Measurement error and the bias

S3.1 A Gaussian benchmark for the gap

Under a Gaussian wealth process the denominator gap has a closed form. Let log wealth $x_s = \log W_s$ be stationary and first-order autoregressive,

$$x_t = \theta x_{t-1} + \varepsilon, \quad x_{t-1} \sim \mathcal{N}(0, \sigma^2), \quad \varepsilon \sim \mathcal{N}(0, \sigma^2(1 - \theta^2)), \quad (\text{S6})$$

so (x_{t-1}, x_t) is bivariate normal with correlation equal to the persistence $\theta \in [0, 1)$, and σ is the standard deviation of log wealth. Ranks are fixed at $\theta = 1$ and turn over more as θ falls. I write ϕ and Φ for the standard normal density and distribution function and z_α for the α -quantile.

Proposition S3.1 (Closed-form wedge). *Under (S6), group d lies between the quantiles $z_{(d-1)/D}$ and $z_{d/D}$ and its denominator wedge is*

$$\frac{\beta_E^d}{\beta_S^d} = \frac{\Phi(\theta\sigma - z_{(d-1)/D}) - \Phi(\theta\sigma - z_{d/D})}{\Phi(\sigma - z_{(d-1)/D}) - \Phi(\sigma - z_{d/D})}, \quad (\text{S7})$$

and for the top group of mass p it is $\beta_E^D / \beta_S^D = \Phi(\theta\sigma - z_{1-p}) / \Phi(\sigma - z_{1-p})$.

Proposition S3.2 (Monotone in persistence). *The top wedge $\Phi(\theta\sigma - z_{1-p}) / \Phi(\sigma - z_{1-p})$ is strictly increasing in θ , equals one at $\theta = 1$, and is below one for $\theta < 1$. The top denominator gap is therefore zero without wealth rank mobility and widens monotonically as persistence falls.*

On the untrimmed matched panel, over the households holding positive wealth in both waves, $\sigma = 2.0$ and $\theta = 0.84$. The closed form then puts the denominator gap at the top wealth decile at -14.7% . The matched panel measures -14.8% under the cross-section ranking the synthetic method uses, and -7.3% re-ranked within the untrimmed matched panel. The benchmark fixes the order of magnitude of the gap and not a point estimate.

S3.2 Wealth rank mobility versus measurement error

Transitory measurement error in recorded wealth disturbs ranks, adds spurious wealth rank mobility and moves the end group's mean lagged wealth toward the population mean. Part of the denominator gap could therefore be noise. Let recorded log wealth be $y_s = x_s + u_s$. The signal x_s is stationary and first-order autoregressive with persistence $\tilde{\theta}$. The noise u_s is classical, independent across waves and of x . The reliability is $\eta = \text{Var}(x) / \text{Var}(y)$. The one-

and two-wave autocorrelations of recorded log wealth are

$$\rho_1 = \text{corr}(y_t, y_{t-1}) = \eta\tilde{\theta}, \quad \rho_2 = \text{corr}(y_t, y_{t-2}) = \eta\tilde{\theta}^2. \quad (\text{S8})$$

so three wealth waves identify both parameters,

$$\tilde{\theta} = \frac{\rho_2}{\rho_1}, \quad \eta = \frac{\rho_1^2}{\rho_2}. \quad (\text{S9})$$

The signal is the persistent component and the noise the transitory one.

On the 16,051 household-waves holding positive wealth in three consecutive waves, $\rho_1 = 0.851$ and $\rho_2 = 0.778$, so $\tilde{\theta} = 0.914$ and $\eta = 0.93$. The wealth process mean-reverts, so the wealth rank mobility is not an artifact of noise. At the de-noised parameters $(\sigma\sqrt{\eta}, \tilde{\theta})$ the Gaussian wedge of Proposition S3.1 puts the top denominator gap at 8.1%. At that subsample's recorded parameters the wedge puts it at 14.0%, so 42% of the gap is noise. Applying that ratio of wedges to the recorded understatement through Equation (4) takes the top understatement from 63% to 36%. With a serially correlated error identified from a fourth wave, the understatement is 32% (Section S5.4).

Selecting the end group on recorded end wealth also places that wave's error in the numerator. Ranking the end group on the following wave's wealth removes that selection. On the 23,957 household-waves observed in three consecutive waves, the top end-group rate is then 12.6% (11.8 to 13.4) against 15.9% under the recorded ranking. The understatement is 112% on this grouping. The end-group gradient falls from 8.8 to 0.4 percentage points (-1.7 to 2.7). Forward ranking also attenuates through genuine wealth rank mobility, so the fall is an upper bound on the noise in the gradient. Ranking the start group on the preceding wave's wealth removes the selection on its own denominator. The top start-group rate is then 6.3% against 3.2%, and the start-group gradient -7.2 percentage points (-9.2 to -5.1) against -17.0, so its negative sign survives.

S4 Data and variable construction

S4.1 Sample and timing

The 12 biennial transitions end between 2001 and 2023, and the first takes its lagged wealth from 1999. A family unit is a reference person plus a partner where one is present. A household enters a transition only when both of its wave records exist.

The sample is the Survey Research Center core that the PSID has followed since 1968. I drop the Survey of Economic Opportunity oversample and the two immigrant refreshers. Reference persons are aged 20 to 65. After the trim the matched panel holds 33,676 household-wave observations. Lagged net worth has no sign filter.¹

S4.2 Net worth, ranking, and denominator

Net worth is total assets minus total liabilities. Assets are the primary residence and other real estate at gross value, stocks, mutual funds and investment trusts, and private business and farm wealth. Assets also include individual retirement accounts, defined-contribution plans, deposit accounts, money-market funds, certificates of deposit, bonds, Treasury bills and collectibles. Liabilities are mortgage debt on the residence and other real estate plus credit-card, student, medical, family and other consumer debt. Vehicle equity is excluded. The PSID's own wealth measure includes it and omits the defined-contribution balances (Cooper, Dynan and Rhodenhiser, 2019). Nothing is top-coded. Bracketed amounts enter at the midpoint of their range.

Table S1 reports the sample in dollars. In the top displayed wealth decile the median net worth is \$1,142,126 and the mean \$1,336,861. Each displayed wealth decile holds 9.9 to 11.6% of the matched panel, because the ranks are cut on the cross-section before matching and trimming.

I rank households into ten wealth deciles on each wave's cross-section, by end wealth for the end-group rate and by start wealth for the synthetic rate's denominator. Both cross-sections are restricted to the age window. The lagged one uses the reference person's age in the later wave, and the 1999 base wave has no age restriction. Households in the wave- t top wealth decile with no usable wave- $(t - 1)$ record drop out of the matched panel. Those households are 15.2% of that wealth decile and hold 14.1% of its wealth. A mid-period denominator would average the start group's start wealth with the end group's end wealth, and the end group is selected on that end wealth.

¹A companion working paper benchmarks the PSID's coverage of the top wealth tail against the Survey of Consumer Finances (Van Langenhove, 2026).

Table S1: The estimation sample in dollars, by displayed wealth decile

Wealth decile	N	Share (%)	Median W_{t-1}	Median W_t	Mean W_t
41–50	3,330	9.9	18,000	26,500	31,124
51–60	3,503	10.4	46,000	62,000	70,534
61–70	3,695	11.0	94,500	122,500	134,412
71–80	3,784	11.2	176,100	226,000	245,234
81–90	3,907	11.6	349,700	446,000	480,601
91–100	3,495	10.4	875,000	1,142,126	1,336,861

Notes: Nominal dollars, pooled over the 12 biennial transitions. W_{t-1} is lagged net worth and W_t end net worth. Share is the wealth decile's percentage of each wave's matched panel, averaged over waves. Sample as in Table 1.

S4.3 The saving numerator

The numerator is $N = W_t - T$. T is the wealth that arrived with members who joined the household, net of the wealth that left with members who departed. For one household the saving rate is

$$s = \frac{W_t - T}{W_{t-1}} - 1 = \frac{(W_t - W_{t-1}) - T}{W_{t-1}}, \quad (\text{S10})$$

and the group rate divides the group mean of $W_t - T$ by the group mean of W_{t-1} (Definition 1). Capital gains stay in.

S4.4 The capital-gains series

Capital gains are the two-year valuation change on the primary residence, other real estate, private business and farm wealth, and equity and retirement holdings. Realized and unrealized gains both count, net of the household's own contributions, withdrawals and improvements. A main home sold within the period books a realized gain of the sale price less the depreciated lagged value. The replacement home starts the next period's base. Section S5.6 removes the entry-wave main-home component, so a first-home purchase is not booked as a gain.

S4.5 Pooling, annualization, and trimming

I normalize each wave by its mean lagged wealth and average the group aggregates over the 12 transitions before taking any ratio. The biennial saving rate is halved. Equation (4) takes the biennial gross ratio R_E^d . On the matched panel I trim the top and bottom one percent of end wealth and of lagged wealth in each wave.

An outlier correction replaces two patterns in each component's household series with the average of the nearest recorded neighbours. One pattern is an isolated zero between two non-

zero waves. The other is a value more than k times its own magnitude away from both neighbours. The threshold k is 250 for equity, fixed-income assets, other assets and non-mortgage debt, and 2,000 for business wealth. For pensions k is 50, for the main home and its mortgage 5, and for other real estate and its mortgage 10.

A third pass for pensions, housing and mortgages replaces a value more than 50 times its own magnitude away from the mean of its three closest non-zero neighbours. That pass runs only when those neighbours lie at most three waves away on median. The isolated-zero rule skips the main home and its mortgage, and no series endpoint is replaced. Both corrections run before the wealth deciles are cut. Without the outlier correction the top understatement is 67% and without the bracket imputation 66%, against 63% in the baseline.

S5 Robustness of the bias

S5.1 The saving bias

Table S3 re-estimates the top understatement and the saving-wealth gradient from the 41 to 50 wealth decile under one change to the construction at a time. The three grouping rows re-rank households within the matched panel into quintiles, wealth deciles and vigintiles, and the wealth-concept rows re-rank them on the alternative concept. The other rows keep the baseline wealth deciles.

The understatement runs from 33 to 71 percent of the end-group rate across the grouping, age window, floor, trim and numerator rows. The end-group gradient stays between 9.5 and 40.7 percentage points. The synthetic gradient never exceeds 7.4 points and turns negative on each of the three grouping rows.

Re-ranked within the matched panel, the understatement is 44 percent and the synthetic saving-wealth gradient changes sign.

Baseline grouping Proposition 2.4 is stated on equal-sized groups, and the baseline end groups hold 5.5 to 13.7% of a wave each. On that grouping the inequality holds at 98% of the 108 pairs in cumulative wealth shares and at 94% in the group means.

Mid-period denominator With the midpoint of start and end wealth as the denominator, the top end group saves 8.7% and the synthetic rate reports 5.9%. The understatement is 33%. On the re-ranked wealth deciles the top denominator gap shrinks from 10.4% to 0.1%, and the cumulative dominance of Proposition 2.4 holds at only 42% of the wave-and-threshold pairs.

Two halves The 12 transitions split into 2001–2009 and 2011–2023. The understatement is 61 percent in the first half and 64 in the second, the end-group gradient 15.4 and 11.9 percentage points, and the synthetic gradient 8.0 and -6.5.

Median saving rates On households with positive lagged wealth the median household saving rate puts the top end group at 15.2% and the top start group at 1.8%. The synthetic rate has no median analog. Across the displayed wealth deciles the immobile households' median saving rate stays between 5.6 and 7.3%. From the 61 to 70 wealth decile the median saving-wealth gradient is -7.8 points ranked on start wealth, 1.7 among the immobile households and 2.3 ranked on end wealth. The aggregate end-group gradient is 9.8.

Table S2: The four saving-wealth gradients under four base wealth deciles and an anchor-free slope

Base decile	End-group		Start-group		Synthetic		Immobile-group	
	pp	90%	pp	90%	pp	90%	pp	90%
41-50	13.6	[10.9, 16.5]	-53.0	[-58.7, -48.0]	0.7	[-0.1, 1.5]	0.8	[-0.3, 2.0]
51-60	12.6	[10.1, 14.9]	-26.5	[-29.0, -24.1]	1.2	[0.6, 1.8]	0.7	[-0.1, 1.6]
61-70	9.8	[7.8, 11.9]	-19.0	[-21.2, -17.1]	1.4	[0.9, 2.0]	1.8	[1.1, 2.5]
71-80	7.6	[6.1, 9.1]	-12.6	[-14.2, -11.1]	1.7	[1.2, 2.3]	2.1	[1.3, 2.8]
<i>Anchor-free: OLS slope across the six displayed deciles, pp per decile</i>								
Slope	2.57	[2.10, 3.03]	-9.22	[-10.08, -8.44]	0.10	[-0.02, 0.22]	0.10	[-0.07, 0.27]

Notes: Each entry is the top wealth decile's saving rate minus the base wealth decile's, in percentage points. Intervals are 90 percent household block bootstrap intervals from 1,000 replicates. The final row is the OLS slope of the saving rate on the wealth decile index across the six displayed wealth deciles, in percentage points per wealth decile. The immobile-group column groups the households whose wealth decile did not change. Sample as in Table 1.

Survey weights The weight is the PSID family longitudinal weight, ER20394 in 2001 through ER85812 in 2023. Weighted, the top understatement is 61%, from 14.2% down to 5.5%. The end-group gradient is 14.6 percentage points against 0.5 synthetic.

Including the oversample Adding the Survey of Economic Opportunity oversample and the two immigrant refreshers takes the matched panel from 30,878 to 49,863 household-waves. The two samples share 11 transitions ending 2021. The top end-group rate is 15.5% against 15.6%, and the understatement 66% against 63%. From the 61 to 70 wealth decile the end-group gradient is 10.3 against 9.8 and the start-group gradient -18.8 against -19.2. From the 41 to 50 wealth decile the end-group gradient is 39.7 against 13.7, because the added households enter that wealth decile.

S5.2 The saving-wealth gradient and its anchor

Section 3.1 anchors the saving-wealth gradient at the 61 to 70 wealth decile. Table S2 measures each of the four saving-wealth gradients from each of the 4 candidate base wealth deciles. Its last row is the OLS slope of the saving rate on the wealth decile index across the six displayed wealth deciles. Five times the slope is the counterpart of an endpoint difference.

The end-group gradient is positive at every anchor and under the slope, and its interval excludes zero. The gradient is 12.6 percentage points from the 51 to 60 wealth decile, 9.8 from the 61 to 70 and 7.6 from the 71 to 80. The start-group gradient is negative throughout, at -26.5, -19.0 and -12.6. The immobile-group gradient is 0.7 (-0.1 to 1.6), 1.8 (1.1 to 2.5) and 2.1 (1.3 to 2.8). Its slope is 0.10 points per wealth decile and the interval -0.07 to 0.27 covers zero.

Per wealth decile the end-group profile rises 2.57 percentage points (2.10 to 3.03) and the synthetic profile 0.10 (-0.02 to 0.22). On the re-ranked equal-sized wealth deciles the end-group rise from the same three anchors is 9.5, 9.9 and 7.4 percentage points and the synthetic fall -9.2, -5.7 and -2.9. Every interval excludes zero. The slopes are 2.45 (1.99 to 2.85) for the end group and -2.77 (-2.91 to -2.60) for the synthetic rate.

The start-group rate falls from 55.5% to 2.6%.

S5.3 Further construction checks

Household composition Restricting the panel to households whose size and marital status do not change drops 23% of it. The top understatement is 60% and the end-group gradient 13.6 percentage points.

Entrants' lagged wealth Under a \$5,000 floor on lagged wealth, 22,485 household-waves remain and the entrants save 69.4% on 11.8% of the top wealth decile's lagged wealth. At \$10,000 and \$25,000 the saving rate is 69.3% and 68.6%. The entrants' contribution to the top end-group rate is 7.2 and 7.2 percentage points under those floors, against 7.6 without one.

Repaired identifiers Following the reference person or partner across an identifier change links 34.1% of the household-waves without a lagged wealth record to their source households and adds 3,980 household-waves. On that wider panel the top end group saves 16.1% against a synthetic 5.7%. The understatement is 64% against 63% in the baseline.

Wealth concept Excluding pension wealth, excluding home equity or counting assets gross of debt leaves the top understatement between 44 and 59%. Each row re-ranks the wealth deciles on the alternative concept within the matched panel.

Age and education Split at age 40 and at completed college, the top understatement stays between 56 and 83% across the four groups and is largest for the households under 40.

Housing tenure Start-wave renters who own at the end wave are 4% of the top wealth decile's upward entrants and hold 1.9% of their lagged wealth. On the 20,456 owner household-waves the understatement is 60% and the end-group gradient 24.1 percentage points. On the 13,220 renter household-waves the synthetic top saving rate is negative, so the understatement is 106%. The renter end-group gradient is -22.4 points.

Wider age window Re-forming the ranks on the unmasked wealth panels, the paper's own window gives an understatement of 63% on 33,817 household-waves. Widening the window

Table S3: The top-group saving bias and the saving-wealth gradient across construction choices

Specification	N	End-gr. s_E^{top} (%)	Synth. $s_{\text{syn}}^{\text{top}}$ (%)	Bias (%)	End-gr. grad. (pp)	Synth. grad. (pp)
Deciles, net numerator, age 20–65	33,676	15.4	5.7	-63	13.6	0.7
<i>Grouping (re-ranked in matched panel)</i>						
Quintiles	33,676	13.7	9.2	-33	9.5	-10.4
Deciles	33,676	15.6	8.8	-44	14.9	-14.2
Vigintiles (top 5%)	33,676	18.2	8.3	-54	17.5	-17.3
<i>Age window</i>						
Heads 25–60	29,636	17.0	5.7	-67	14.8	0.4
Heads 30–55	22,332	18.1	5.3	-71	17.6	0.3
<i>Floor</i>						
Lagged wealth $\geq$ \$5k	22,485	15.1	5.8	-62	29.9	-1.0
Lagged wealth $\geq$ \$10k	21,414	15.1	5.8	-62	32.7	-0.7
Lagged wealth $\geq$ \$25k	19,140	15.0	5.8	-61	40.7	7.4
<i>Trim</i>						
Extra 0.5% trim	33,071	16.0	5.9	-63	14.9	0.8
Extra 2.5% trim	31,128	19.7	6.4	-68	20.4	1.4
<i>Numerator</i>						
Gross change (no transfer net)	33,676	15.5	5.8	-63	13.4	0.5
Gift and bequest receipts netted	33,676	14.4	4.8	-66	14.3	1.6
<i>Household composition</i>						
Stable size and marital status	25,931	15.4	6.1	-60	13.6	0.7
<i>Wealth concept</i>						
Excluding pension wealth	23,848	17.0	8.1	-52	14.6	-4.8
Excluding home equity	22,017	20.9	8.5	-59	33.5	3.0
Assets gross of debt	30,758	14.0	7.9	-44	8.0	-3.4
<i>Age band</i>						
Heads under 40	14,199	33.4	5.7	-83	19.6	1.6
Heads 40 and over	19,477	13.8	6.0	-56	20.6	0.9
<i>Education</i>						
College (16+ years)	11,403	14.8	6.1	-59	8.5	1.0
Below college	21,535	16.3	4.8	-71	16.2	-0.7
<i>Aggregation</i>						
Median of ratios	25,117	15.2	1.8	-88	8.3	-15.4
<i>Weighting</i>						
Core and immigrant family weight	33,515	14.2	5.5	-61	14.6	0.5

Notes: Each row repeats the estimation under one change. Columns report the sample size, the end-group and synthetic rates at the top wealth decile, the understatement, and the two saving-wealth gradients from the 41 to 50 wealth decile. Saving rates and the understatement are annualized and in percent, and gradients are in percentage points. The first row is the baseline. In the median row the synthetic columns report the start-group rate. Otherwise as in Table 1.

to age 75 gives 58% on 38,441, and dropping it gives 57% on 41,631. The end-group gradient stays between 13.4 and 15.5 percentage points. From the 61 to 70 wealth decile the start-group gradient stays between -15.7 and -19.1.

Age bands Re-ranked within the age bands 20 to 34, 35 to 49 and 50 to 65, the share of immobile households is 40% against 45% under the pooled ranking. The top own-band end group

saves 19.2% and the synthetic rate reports 8.8%. The understatement is 54%. The end-group and start-group gradients are 22.3 and -21.3 percentage points on 33,655 household-waves.

Longer horizons Re-ranked within the matched panel, the top denominator gap is 10.4% at two years, 14.9% at four and 18.8% at six (Table S4). The understatement is 44%, 38% and 36%.

Attrition I weight the panel by the inverse of a retention probability from a logit of presence in wave t on log lagged wealth, age and age squared. The top understatement is 61% and the end-group gradient 25.3 percentage points (Table S4). The logit runs on 23,214 household-waves and drops the 2001 transition. Unweighted on those household-waves, the understatement is 61% and the end-group gradient 26.2 points.

The dollar trim Without the trim, 34,701 household-waves remain. The top end group saves 13.8%, the start group 5.2%, the immobile households 9.1% and the synthetic rate reports 5.7%. The untrimmed immobile-group rate normalises each wave on the immobile households alone. The understatement is 59% against 63% in the baseline, and 60% at half the trim. The trim drops 12% of the top wealth decile's households, and those households hold 45% of its lagged wealth. Trimming lagged wealth alone keeps 33,997 household-waves. There the understatement is 48% and the end-group gradient 19.8 points against 6.2 synthetic.

Weights on shared waves On the 11 waves shared with the full-sample build the weighted top understatement is 62% and the unweighted one 63%. The full sample's weight records reach only 51% of its households, and its unweighted understatement is 66%.

Processing switches On the 11 waves all three builds hold, the understatement is 63% in the baseline, 67% without the outlier correction and 66% without the bracket imputation.

Re-formed ranks I re-form the ranks and the trim inside each of 1,000 bootstrap replicates. The top understatement is 63% (59 to 66) against 60 to 66 with the ranks held fixed, and the end-group gradient 13.6 points (11.7 to 18.0).

Single transitions Two sets of non-overlapping transitions put the understatement at 65% (60 to 70) and 61% (56 to 66). Drawing one transition per household gives 67% (59 to 77) on 8,452 households and across 20 such draws the point runs from 59 to 75%.

Leave-one-out Dropping each of the 12 transitions in turn, the top understatement runs 60.4 to 65.6% around the baseline 63%. Removing the transition ending in 2021 moves it most, to

Table S4: The top-group saving bias at longer horizons and under attrition weighting

Specification	N	End-gr. s_E^{top} (%)	Synth. $s_{\text{syn}}^{\text{top}}$ (%)	Bias (%)	End-gr. grad. (pp)	Synth. grad. (pp)
<i>Horizon</i>						
Four-year transition	17,838	16.4	10.2	-38	11.0	-4.5
Six-year transition	13,922	17.8	11.3	-36	10.2	-4.0
<i>Attrition</i>						
Inverse-retention weighted	23,214	14.7	5.8	-61	25.3	-0.3

Notes: Columns as in Table S3. The horizon rows re-rank within the matched panel, so their two-year reference is the re-ranked row of Table S3. The attrition row weights households by the inverse of a retention probability fitted on lagged wealth and age. Otherwise as in Table 1.

65.6%. The delete-one jackknife puts a standard error of 5.35 points on the understatement, against 5.5 for the wave bootstrap and 1.74 for the household bootstrap.

Repeated cross-section numerator I form the numerator on the full wave- t cross-section and the denominator on the full preceding one, each trimmed on its own wealth. The top synthetic rate is then 4.9% against 5.7% on the matched panel, and the understatement 69% against 63%. The cross-section adds back the 15.2% of the wave- t top wealth decile with no lagged record, holding 14.1% of its wealth.

S5.4 Serially correlated measurement error

Section S3.2 treats the measurement error as classical. Let the error u_s instead follow a stationary first-order autoregression of persistence ρ_u , independent of the signal. The autocorrelations of recorded log wealth are then

$$\rho_k = \eta \tilde{\theta}^k + (1 - \eta) \rho_u^k, \quad k = 1, 2, 3, \quad (\text{S11})$$

and classical noise is the case $\rho_u = 0$. A fourth wave supplies ρ_3 , and the three autocorrelations identify $(\tilde{\theta}, \rho_u, \eta)$.

On the 11,139 four-wave household-waves, $\rho_u = 0.10$. The de-noised top understatement is 32% against 35% under the classical identification on the same waves. The case in which the signal has unit persistence and all mean reversion is noise predicts a six-year autocorrelation of 0.77. The measured six-year autocorrelation is 0.75.

Bootstrap interval Resampling households and recomputing both the recorded understatement and the ratio of Gaussian wedges, over 1,000 replicates, puts the de-noised understatement at 36% (32 to 40).

Table S5: Total and active saving rates out of wealth, by wealth decile

Wealth decile	Total saving		Active saving		Active gap	
	End-group s_E^d	Synthetic s_{syn}^d	End-group s_E^d	Synthetic s_{syn}^d	pp	% of $ s_E^d $
41-50	1.5	5.5	1.0	4.9	3.9	374
51-60	2.5	4.8	-0.6	1.5	2.1	343
61-70	5.0	4.2	1.3	0.5	-0.7	-58
71-80	7.3	4.0	3.8	0.8	-3.0	-80
81-90	9.0	4.7	4.1	0.1	-4.0	-98
91-100	15.2	5.6	5.1	-3.0	-8.1	-159

Notes: The active numerator subtracts the capital gain from the change in net worth. The capital gain is the two-year valuation change on housing, other real estate, private business and farm wealth, and equity and retirement holdings. The gain is net of the household's own contributions, withdrawals and improvements. Saving rates are annualized and in percent. The active gap is the synthetic active rate less the end-group active rate, in percentage points and as a percentage of the end-group rate's magnitude. Levels are not comparable with Table 1. PSID 2001 to 2023, subsample where capital gains are observed.

Resampling waves Resampling the 12 waves instead of households, over 2,000 replicates, gives the top understatement a standard error of 5.5 points and an interval of 55 to 73. The household bootstrap gives 1.7 and 60 to 66. The synthetic rate understates the end-group rate in all 12 waves. Across the waves it runs from 6.5 to 12.9 percentage points. It averages 9.1 over the 2007 to 2009 and 2019 to 2021 transitions, against 9.9 over the other ten.

S5.5 The denominator gap above the top wealth decile

The gap widens as the group narrows, because more of its members are upward entrants. Re-ranked within the untrimmed matched panel, the top denominator gap is 7.3% at the top wealth decile, 9.1% at the top five percent and 14.3% at the top wealth percentile. The upward-entrant share rises from 22% to 37%. The top wealth percentile holds only 347 matched household-wave observations.

S5.6 Active saving, net of capital gains

The active numerator is $W_t - T - G$, with G the household's capital gains over the transition (Table S5). The subsample is the 31,661 household-waves where capital gains are observed, and the total rate is recomputed on it. The capital-gains series of Section S4.4 enters without its entry-wave main-home component.

The top end group's active rate is 5.1%, or 33% of its total rate (30 to 37% over 1,000 resamples). The active understatement exceeds the total one, because the denominator gap is the same and the saving rate is smaller (Proposition 2.3).

The end group's active saving-wealth gradient from the lowest displayed wealth decile is 4.0 percentage points (1.2 to 6.7) against -7.9 in the synthetic method (-10.0 to -5.9). From the 61 to 70 wealth decile the two are 3.8 (2.2 to 5.5) and -3.5 (-4.6 to -2.4). From the 51 to 60 wealth decile they are 5.7 (3.6 to 7.8) and -4.5 (-5.9 to -3.1). The end-group gradient is positive and the synthetic gradient negative at every anchor.

On the 22,680 household-waves with a third wave and observed gains, ranking the end group on the following wave's wealth puts its top active rate at 4.3% (3.7 to 5.0). Under the recorded ranking the rate is 5.5%. The active end-group gradient from the 61 to 70 wealth decile falls from 2.7 to -2.2 percentage points (-4.2 to -0.2). The total gradient on the same household-waves falls from 9.4 to 0.5. Forward ranking also attenuates through genuine wealth rank mobility, so the fall is an upper bound on the noise.

Entry-wave correction Without the entry-wave correction the lowest displayed wealth decile's active rate is -29.2% rather than 1.0%. That level is an artifact of first-home purchases. At the top wealth decile the uncorrected end-group active rate is 4.6% and the synthetic rate -3.4%. The understatement is 173% (155 to 195%), and 30% of the total rate survives (27 to 34%). The uncorrected synthetic active saving-wealth gradient is positive at every anchor. From the lowest displayed wealth decile it is 24.2 (20.6 to 27.9). From the 51 to 60 wealth decile it is 8.6 (6.6 to 10.6), and from the 61 to 70 2.4 (1.1 to 3.7). The uncorrected end-group gradient is 33.8 (30.7 to 37.0), 18.1 (16.3 to 20.0) and 9.7 (8.3 to 11.4). The reversal of the synthetic active gradient therefore rests on the correction.

S6 A fixed-grouping placebo

The synthetic method is consistent for a fixed grouping, because its two cross-sections then sample the same population group (Deaton, 1985).

S6.1 Construction

For a transition ending at wave t , the synthetic numerator is the mean of end wealth net of transfers over the wave- t cross-section in group k . The denominator is the mean of lagged wealth over the wave- $(t - 2)$ cross-section in the same group. The end-group rate groups the matched households present in both waves by their end-wave label. I normalise each transition by the base cross-section's mean wealth and weight the waves equally. The synthetic arm runs on the 33,369 pooled household-waves of the cross-sections and the panel arm on their matched subset. The placebo keeps only households with positive net worth, trims on end wealth alone, re-ranks the wealth deciles inside each trimmed wave and starts from the 2001 base wave.

S6.2 Fixed grouping versus wealth rank

Table S6 reports three groupings, two mobile and one fixed. Under wealth deciles the end-group profile rises 12.3 percentage points from the bottom displayed wealth decile to the top and the synthetic profile 0.0. The two profiles correlate 0.15. The synthetic rate understates the top wealth decile by 67% of the end-group rate, against 63% on the matched panel (Section 3.3).

Under birth cohort the end-group gradient from the oldest cohort to the youngest is 23.9 percentage points and the synthetic gradient 15.1 (Table S7). The synthetic gradient retains 63% of the end-group gradient. The two profiles correlate 0.99. Under education the end-group gradient is 1.5 percentage points and the synthetic gradient 0.2.

S6.3 Turnover without alignment

Income rank turns over between waves while the denominator stays lagged wealth. I form the income deciles within each wave on the same trimmed cross-section, on the households with a recorded income measure. Across income deciles the end-group profile rises 25.0 percentage points and the synthetic profile -1.6 (-3.2 to -0.1, Table S8). The two profiles correlate -0.59. At the top income decile the synthetic rate understates the end-group rate by 85% (83 to 88%). Proposition 2.3 covers this grouping and Proposition 2.4 does not.

Table S6: End-group and synthetic saving-wealth profiles under three groupings

Grouping	End-group gradient (pp)	Synthetic gradient (pp)	Profile corr.	Mean $ s_{\text{syn}}^d - s_E^d $ (pp)
Wealth deciles (time-varying, is the denominator)	12.3	0.0	0.15	4.3
Income deciles (time-varying, not the denominator)	25.0	-1.6	-0.59	8.2
Birth cohorts (fixed, not the denominator)	23.9	15.1	0.99	3.2

Notes: Columns report the end-group and synthetic saving-wealth gradients in percentage points, the correlation of the two profiles across groups and their mean absolute gap. A gradient is the top group minus the bottom group. Wealth and income deciles are re-ranked within each wave while birth cohorts never change. Lagged wealth is the denominator throughout. Separate cross-sections of the PSID waves 2001 to 2023.

S6.4 Inference

Intervals are 90 percent percentile intervals from 1,000 replicates of a household block bootstrap that re-forms both cross-sections and the within-wave ranks in each replicate. For the wealth arm the end-group gradient lies between 10.5 and 14.7 and the synthetic gradient between -1.4 and 1.8. The top-group understatement runs from 55 to 75%. For birth cohorts the synthetic gradient stays between 10.1 and 20.7 percentage points and the profile correlation above 0.84 in 95% of replicates.

Table S7: End-group and synthetic saving rates by birth-cohort decade

Birth cohort	End-group s_E^d (%)	Synthetic s_{syn}^d (%)	Gap (pp)
1940s	5.4	5.1	-0.3
1950s	6.1	5.9	-0.2
1960s	9.0	9.8	0.8
1970s	14.1	10.9	-3.2
1980s	21.9	16.4	-5.5
1990s	29.4	20.3	-9.1

Notes: Saving rates are out of lagged wealth, annualized and in percent. Birth cohort is fixed, so a household never changes group. Separate cross-sections of the PSID waves 2001 to 2023.

Table S8: End-group and synthetic saving rates out of wealth by income decile

Income decile	End-group s_E^d (%)	Synthetic s_{syn}^d (%)	Gap (pp)
41-50	1.8	5.6	3.8
51-60	5.1	4.1	-0.9
61-70	8.5	4.1	-4.3
71-80	10.1	3.8	-6.4
81-90	14.5	3.9	-10.7
91-100	26.8	3.9	-22.9

Notes: Saving rates are out of lagged wealth, annualized and in percent. Income deciles are ranked within each wave. Income is total inflows net of taxes and includes capital gains and inheritances. The displayed window matches Table S6. Separate cross-sections of the PSID waves 2001 to 2023.

S7 The correction in published work

Saez and Zucman (2016) report a synthetic saving rate by wealth fractile. Table S9 compares it with the end-group rate on its PSID counterpart.

S7.1 The published synthetic saving rate

Saez and Zucman define synthetic saving S^p for fractile p by $W_{t+1}^p = (1 + q_t^p)(W_t^p + S_t^p)$ and report $s_t^p = S_t^p / Y_t^p$. The numerator pairs the end-date fractile's wealth with the start-date fractile's as in Definition 1, and the denominator is the start-date fractile's income. Proposition S1.4 signs the bias under that denominator only under conditions Section S1.4 does not verify. Proposition 2.3 transfers through the ratio of the end group's flow to the synthetic flow. The $(1 + q_t^p)$ term nets out asset price changes, so the comparable numerator is active saving (Section S5.6).

S7.2 Active saving at the top wealth percentile

Synthetic active saving at the top wealth percentile is -4.4% a year against 3.2% for the end group on the same households and the same numerator. The household block bootstrap puts the synthetic rate between -7.7 and -1.6 and the end-group rate between 0.6 and 5.8 . The percentile holds 325 household-waves.

Netting out the reported gift and bequest receipts leaves the end-group rate at 2.6% (0.1 to 5.2) and the synthetic rate at -4.9% (-8.3 to -1.9). Resampling the 12 waves instead of the households over 2,000 replicates gives 0.4 to 5.7 for the end group and -6.4 to -2.3 for the synthetic rate. The end-group rate is the higher of the two in 12 of the 12 waves.

Above the top wealth decile the synthetic active rate is -1.3% for the top five percent and -3.1% for the top two while the end-group rate stays near 3.5% . At the wealth decile itself the synthetic rate is -0.4% on an interval that reaches 0.6 and so does not exclude zero.

S7.3 The income-denominated wealth decile profile

On the 27,531 household-waves with an after-tax income measure, 82% of the matched panel, the top wealth decile's end-group flow is 63.6% of its income against a synthetic 28.8%. The published saving rates divide by pre-tax income. The end-group gradient of 63.2 percentage points compresses to 25.9 in the synthetic method. On this subsample the wealth-denominated understatement is 61% against 63% on the full panel.

Table S9: End-group and synthetic active saving rates, top wealth percentile and top five percent

	End-group	Synthetic	Observations
Top wealth percentile	3.2 [0.6, 5.8]	−4.4 [−7.7, −1.6]	325
Top five percent	3.5 [2.1, 4.8]	−1.3 [−2.6, −0.1]	1,629

Notes: Active saving rates out of lagged wealth, annualized and in percent, with 90 percent household block bootstrap intervals from 1,000 replicates in brackets. The end-group column groups households on end wealth and the synthetic column divides the same flow by the start group's lagged wealth. The fractiles are cut on the untrimmed matched panel within the subsample where capital gains are observed, so levels are not comparable with Table 1. PSID 2001 to 2023.

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